Comments on "Spin Connection Resonance in Gravitational General Relativity"

GERHARD W. BRUHN[†]
FRIEDRICH W. HEHL[‡]
ARKADIUSZ JADCZYK[§]

(Received 29 July 2007, file Evans-Resonance06.tex)

We comment on the recent article of Evans in this journal [1]. We point out that the equations underlying Evans' theory are highly problematic. Moreover, we demonstrate that the so-called "spin connection resonance", predicted by Evans, cannot be derived from the equation he used. We provide an exact solution of Evans' corresponding equation and show that is has definitely *no* resonance solutions.

PACS numbers: 03.50.Kk; 04.20.Jb; 04.50.+h

Keywords: Electrodynamics, gravitation, Einstein-Cartan theory, Evans' unified field theory

1. Introduction

Over the last years, Evans' papers deal mainly with his so-called Einstein-Cartan-Evans (ECE) theory, which exists also under the former name "Generally covariant unified field theory" [2]. Evans aims at a fundamental unified field theory for physics. However, a long list of serious errors in his theory is well-known, see [3–7]. Evans never tried to take care of these errors and to improve his theory correspondingly. In fact, he believes that his theory is flawless.

In our opinion it is clear that Evans' theory has been disproved already and is untenable, both from a physical and a mathematical point of view. Nevertheless, he continues to publish papers and to predict new physical effects. In [1], Evans

[†] Department of Mathematics, Darmstadt University of Technology, Schlossgartenstr. 7, 64289 Darmstadt, Germany; email: bruhn@mathematik.tu-darmstadt.de

[‡] Institute for Theoretical Physics, University of Cologne, 50923 Köln, Germany, and Department of Physics and Astronomy, University of Missouri-Columbia, Columbia, MO 65211, USA; email: hehl@thp.uni-koeln.de

[§] Center CAIROS, Laboratoire de Mathématiques Emile Picard, Institut de Mathématiques de Toulouse, Université Paul Sabatier, 31062 TOULOUSE CEDEX 9, France; email: arkadiusz.jadczyk@cict.fr

foresees a new "spin connection resonance" (SCR) effect. The aim of our article is to take a critical view on [1].

In Sec.2 we go through Evans' article [1] and point out numerous mistakes and inconsistencies in the set-up of his theory. Most of it is known from the literature [3–7]. In Sec.3 we turn to the new SCR effect, which Evans derives from a certain ordinary differential equation of second order. Even though the derivation of this equation is dubious, we start from exactly the same equation as Evans did and prove that this equation has no resonance type solutions as Evans claims. This shows that Evans' SCR effect is a hoax.

2. General comments on Evans' paper

The paper [1] deals with what the author calls 'Cartan geometry'. The term is not defined in the paper, so the reader has to guess what the exact meaning is of this term. From the content of the paper it seems plausible that the term means: linear connection in the tangent bundle of a four–dimension manifold, compatible with the metric of Minkowskian signature, see also [7] for a discussion of this 'Riemann-Cartan geometry'. The connection may admit torsion, and the method used is that of Cartan's moving frame (also known as tetrad or vierbein). In what follows we will assume this interpretation of the term 'Cartan geometry' in our paper. We will refer to the equations in Ref. [1] by using double parenthesis.

2.1. Curvature and torsion

Evans' paper starts with what the author calls "the second Cartan structure equation",

$$R^a_{\ b} = D \wedge \omega^a_{\ b}\,, \qquad ((1))$$

and with the second Bianchi identity,

$$D \wedge R^a_{\ b} := 0\,. \qquad ((2))$$

The symbol $D\wedge$ stands, in Evans' notation, for the exterior covariant derivative, ω and R are the connection and the curvature forms, respectively. Eq.((1)) represents the *definition* of the curvature form. The second structure equation, which follows immediately from ((1)) and from the definition of D, is given as

$$R^a_{\ b} = d \wedge \omega^a_{\ b} + \omega^a_{\ c} \wedge \omega^c_{\ b}\,. \qquad ((5))$$

The second Bianchi identity follows from ((5)) by exterior differentiation:

$$d \wedge R^a_{\ b} + \omega^a_{\ c} \wedge R^c_{\ b} - R^a_{\ c} \wedge \omega^c_{\ b} := 0\,. \qquad ((6))$$

Torsion is introduced according to

$$T^a = d \wedge q^a + \omega^a_{\ b} \wedge q^b\,, \qquad ((7))$$

with the tetrad one-forms q^a, which we interpret, according to the context, as a local *orthonormal* coframe.

2.2. Objections to the 'derivation' of Eqs. ((11)) and ((13))

Subsequently Evans writes:

"... Eq.((6)) can be rewritten as

$$d \wedge R^a{}_b = j^a{}_b \,, \tag{(10)}$$

$$d \wedge \widetilde{R}^a{}_b = \widetilde{j}^a{}_b \,, \tag{(11)}$$

where

$$j^a{}_b = R^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge R^c{}_b, \tag{(12)}$$

$$\widetilde{j}^a{}_b = \widetilde{R}^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge \widetilde{R}^c{}_b. \tag{(13)}$$

The tilde denotes the Hodge dual [1–20] of the tensor valued two–form

$$R^a{}_{b\mu\nu} = -R^a{}_{b\nu\mu} \;\ldots\text{"} \tag{(14)}$$

While it is true that ((10)) and ((12)) are a rewriting of ((6)), this is *false* for ((11)) and ((13)). Eqs.((11)) and ((13)) do not follow from differential geometry. Especially the combination of ((11)) and ((13)), namely

$$d \wedge \widetilde{R}^a{}_b = \widetilde{R}^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge \widetilde{R}^c{}_b \,,$$

cannot be derived from the second Bianchi identity ((6)) and does not hold in general. Indeed, $D \wedge R^a{}_b = 0$ does *not* imply $D \wedge \widetilde{R}^a{}_b = 0$, since taking the Hodge dual doesn't commute with D.

2.3. The electromagnetic sector of Evans' theory, the index type mismatch

Eqs.((17)) and ((18)) relate, according to Evans, a generalized electromagnetic field strength F^a and a potential A^a to the torsion and the tetrad, respectively,

$$F^a = A^{(0)} T^a \,, \tag{(17)}$$

$$A^a = A^{(0)} q^a \,, \tag{(18)}$$

where $A^{(0)}$ is, presumably, a universal constant. Evans' next but one equation is the first Bianchi identity,

$$d \wedge T^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \,. \tag{(20)}$$

Let us look at Evans' motivation for his choices ((17)) and ((18)). Evans supposed an analogy of A^a and F^a with the Maxwellian potential one–form A and the field strength two–form F according to

$$A \to A^a \,, \qquad F \to F^a \,. \tag{1}$$

In Maxwell's theory, $F = d \wedge A$ is then put in analogy to Cartan's first structure equation (definition of the torsion) $T^a = D \wedge q^a$.

One serious objection is based on the fact that Evans has not given any information about the relations between the concrete electromagnetic fields $F = (\mathbf{E}, \mathbf{B})$ in physics and his quadruple of two–forms F^0, F^1, F^2, F^3 and the associated quadruple of one–forms A^0, A^1, A^2, A^3. Evans himself ignores that problem of attaching a superscript a to all electromagnetic field quantities without giving a satisfying explanation of that index surplus.

Evans' attempts to interpret (1) appropriately doesn't even work in the case of a simple circularly polarized plane (cpp) wave. His considerations are contradictory and incomplete, and we see no way to define F^0, F^1, F^2, F^3 and A^0, A^1, A^2, A^3 even for a bit more complicated field as, e.g., a superposition of different cpp waves travelling in different directions. This is not a mathematical error, but a physical gap, and we doubt that one can find a general solution of that problem. Anyway, Evans never presented such a solution.

Therefore, Evans' analogy $F \leftrightarrow T^a$, for $a = 0, 1, 2, 3$, causes a *type mismatch* between the *vector* valued torsion two–form T^a and the *scalar* valued electromagnetic field strength two–form F. The analogous holds for $A \leftrightarrow q^a$, for $a = 0, 1, 2, 3$ as well.

Evans' whole SCR paper is based on the dubious assumption that (1), and thus ((17)) and ((18)), make sense in physics. Without a concrete physical interpretation of (1), Evans' whole SCR paper is null and void, regardless whether there are other (mathematical) errors or not.

Moreover, as it was with the second Bianchi identity, so here, Evans' equations ((23)),((16)), and ((17)), if combined, lead to

$$d \wedge \widetilde{T}^a = \widetilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \widetilde{T}^b \,. \tag{2}$$

Eq.(2), contrary to Evans' statement, is *not* a consequence of the first Bianchi identity and does not hold in Cartan's differential geometry. It represents an additional ad hoc assumption.

2.4. The gravitational sector of Evans' theory, objections to Eq.((30))

Eq.((29)) is the field equation of Einstein's general relativity theory,

$$G^{\mu\nu} = kT^{\mu\nu} \,, \qquad ((29))$$

after which Evans writes:

"... Eq.(29) is well known, but much less transparent than the equivalent Cartan equation

$$\begin{aligned} D \wedge \omega^a_{\ b} &= kD \wedge T^a_{\ b} \\ &:= 0 \ldots \text{''} \end{aligned} \tag{(30)}$$

Eq.((30)) is certainly *not* equivalent to ((29)), and it cannot be a part of general relativity theory, be it tensorial or in Cartan form. The reason is very simple: $T^a_{\ b}$ in ((30)) has to be a one–form. Therefore it should be integrated over a world–line and not over a hypersurface of four–dimensional spacetime, as it is done with the energy–momentum tensor. In other words, Eq.((30)) is simply incorrect since the energy–momentum in exterior calculus is a covector–valued three–form (or, if its Hodge dual is taken, a *covector–valued* one–form).

2.5. The wrong 'curvature vector' and the dubious potential equation

Now Evans turns to the combined equation ((5)) and ((10)),

$$d \wedge (d \wedge \omega^a_{\ b} + \omega^a_{\ c} \wedge \omega^c_{\ b}) = j^a_{\ b}\,, \tag{(31)}$$

with his comment that *in vector notation* it gives, in particular,

$$\nabla \cdot \mathbf{R}(\text{orbital}) = \mathbf{J}^0, \tag{(32)}$$

with

$$\mathbf{R}(\text{orbital}) = R^0_{\ 1}{}^{01}\mathbf{i} + R^0_{\ 2}{}^{02}\mathbf{j} + R^0_{\ 3}{}^{01}\mathbf{k}\,. \tag{(33)}$$

It is evident that ((32)) is not equivalent to ((31)), if only for the simple reason that ((31)) involves a three–form, where all indices must be different from each other, while ((32)), with the divergence operator, involves summation over repeated indices. In ((37)), Evans evidently attempts to calculate the $(0i)$ component of the curvature form:

$$\boldsymbol{R}^a_{\ b} = -\frac{1}{c}\frac{\partial \boldsymbol{\omega}^a_{\ b}}{\partial t} - \boldsymbol{\nabla}\omega^{0a}{}_{\ b} - \omega^{0a}{}_c \boldsymbol{\omega}^c_{\ b} + \boldsymbol{\omega}^a_{\ c}\omega^{0c}{}_{\ b}\,. \tag{(37)}$$

This is again incorrect. In fact, starting from ((5)), the calculation of the components $(R_{0i})^a_{\ b}$, for $i = 1, 2, 3$, yields

$$(R_{0i})^a_{\ b} = \partial_0(\omega_i)^a_{\ b} - \partial_i(\omega_0)^a_{\ b} + (\omega_0)^a_{\ c}(\omega_i)^c_{\ b} - (\omega_i)^a_{\ c}(\omega_0)^c_{\ b}\,. \tag{3}$$

Raising the index 0 of ω_0 in the term $\partial_i(\omega_0)^a_{\ b}$, as Evans does, is illegitimate, because the metric component g^{00} of the Schwarzschild metric, which Evans considers, is *not* a constant function of the variables x^i. The sign in front of the time derivative in ((37)) is also wrong.

Then in ((42)), when restricting to the static case, Evans ‘forgets’ one of the quadratic terms of his erroneous ((37)):

$$\boldsymbol{R}^a{}_b = -\boldsymbol{\nabla}\omega^{0a}{}_b + \boldsymbol{\omega}^a{}_c\omega^{0c}{}_b\,. \tag{(42)}$$

Again, this is wrong, since now $\boldsymbol{R}^a{}_b$ is not in the Lie algebra of the Lorentz group. The same error applies to ((44)), where $\omega^{0a}{}_b$ is substituted by $\Phi^a{}_b$,

$$\boldsymbol{R}^a{}_b = -\boldsymbol{\nabla}\Phi^a{}_b + \boldsymbol{\omega}^a{}_c\Phi^c{}_b\,. \tag{(44)}$$

Then Evans adds:

“…It is convenient to use a negative sign for the vector part of the spin connection, so

$$\boldsymbol{R}^a{}_b = -\left(\boldsymbol{\nabla}\Phi^a{}_b + \boldsymbol{\omega}^a{}_c\Phi^c{}_b\right)\ldots\text{”} \tag{(45)}$$

This is another evident and grave error. Since the sign of the connection form is not a question of ‘convenience’ in the theory of gravity, where the curvature tensor contains both linear and quadratic terms in the connection. Changing the sign of the connection forms changes its curvature in an essential way.

Using incomprehensible and sometimes evidently wrong reasonings, such as skipping one term when going from ((37)) to ((42)), as we saw above, Evans postulates a potential equation ((63)) for an unidentified variable Φ for the case of the Schwarzschild geometry. We shall discuss the “electromagnetic analogue of Eq.(63)”, namely Eq. ((65)), in the following section.

3. The Resonance Catastrophe

In the lines after ((31)), Evans writes:

“It is shown in this section that Eq.(31) produces an infinite number of resonance peaks of infinite amplitude in the gravitational potential [2–20]. To show this numerically, Eq.(31) is developed in vector notation…”

This is an unfounded claim followed by *no proof* and no numerical results either. In addition the claim is erroneous as we shall see below. At the very end of his article Evans at last arrives at the topic ‘resonance’ that is already announced in the title of his paper. He reports:

“…The electromagnetic analogue of Eq.(63) is

$$\frac{\partial^2\phi}{\partial r^2} - \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\phi = -\frac{\rho(0)}{\epsilon_o}\cos(\kappa r) \tag{(65)}$$

which has been solved recently using analytical and numerical methods [2–20]. These solutions for ϕ and Φ show the presence of an infinite number of resonance peaks, each of which become infinite in amplitude at resonance.”

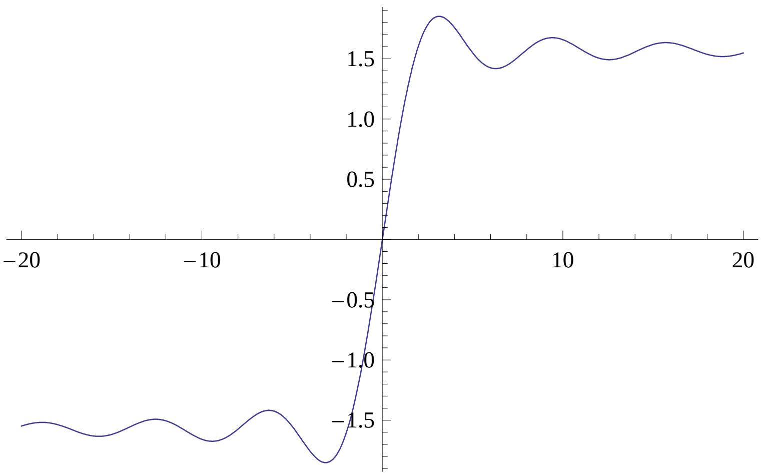

Fig. 1. Graph of the sine integral $\mathrm{Si}(z)$.

Evans' efforts (together with H. Eckardt) with respect to the resonance of ((65)) are available on his website. He attempts to find values of the parameter κ that yield resonances of the right hand side of ((65)) with the eigensolutions of this Euler type ordinary differential equation (ODE). However, the eigensolutions of the associated homogeneous ODE are well-known. The eigenspace is spanned by the special solutions

$$\phi_1 = r \qquad \text{and} \qquad \phi_2 = r \log r \,. \tag{4}$$

Resonance means that the driving term $\cos(\kappa r)$ belongs to the eigenspace, i.e., is a linear combination, with constant coefficients, of the functions ϕ_1 and ϕ_2 for any value of the parameter κ. Obviously this is not the case.

Moreover, the general solution of ((65)) can be calculated. With the help of Mathematica, we obtain

$$\phi(r) = c_1 r + c_2 r \log r - \frac{\rho(0)}{\epsilon_o} \frac{r}{\kappa} \mathrm{Si}(\kappa r) \,, \tag{5}$$

where Si denotes the *sine integral* function defined by

$$\mathrm{Si}(z) := \int_0^z \frac{\sin t}{t} dt \tag{6}$$

for real z satisfying the estimate

$$|\mathrm{Si}(z)| \leq \min(|z|, 2) \,. \tag{7}$$

The graph of $\mathrm{Si}(z)$ is displayed in Fig.1.

Thus, the κ dependent part of the solution (5) satisfies the estimate

$$\left| \frac{\rho(0)}{\epsilon_o} \frac{r}{\kappa} \operatorname{Si}(\kappa r) \right| \leq \frac{\rho(0)}{\epsilon_o} \min(r^2, \frac{2r}{\kappa}) \,. \tag{8}$$

Consequently, the general solution of (65) is bounded for all real values of κ and r. For no value of κ, we will have a resonance of the right-hand-side of ((65)) with the eigensolutions (4).

However, Evans & Eckardt apply a lot of their specific 'new math': an inadmissible rotation of the complex plane of eigenvalues by an angle of 90° and multiplication by the imaginary unit i, among other peculiarities, see [4] for details. Evans & Eckardt succeed in detecting resonance peaks, unattainable to all who are using standard mathematics only.

There are no resonance peaks at all, quite apart from the errors in Evans' theory previous to his equations ((63)) and ((65)).

REFERENCES

[1] M.W. Evans, *Spin connection resonance in gravitational general relativity,* Acta Physica Polonica **B38** (2007) 2211–2220

[2] M.W. Evans, *Generally Covariant Unified Field Theory:* The Geometrization of Physics. Vols.1 to 5, Abramis Academic (2005,2006,2007); see also http://www.aias.us/ and http://www.atomicprecision.com/

[3] G.W. Bruhn, *Remarks on Evans/Eckardt's Web-Note on Coulomb Resonance*, http://www.mathematik.tu-darmstadt.de/ bruhn/RemarkEvans61.html

[4] G.W. Bruhn, *web articles*, http://www.mathematik.tu-darmstadt.de/ bruhn/GCUFT.html

[5] W.A. Rodrigues, Jr. and Q.A. Gomes de Souza, *An ambiguous statement called 'tetrad postulate' and the correct field equations satisfied by the tetrad fields,* Int. J. Mod. Phys. **D14** (2005) 2095-2150; http://arXiv.org/math-ph/0411085

[6] A. Jadczyk, *Remarks on Evans' "Covariant Derivatives"*, http://www.mathematik.tu-darmstadt.de/ bruhn/byArk260107.html

[7] F.W. Hehl, *An assessment of Evans' unified field theory I,* F.W. Hehl and Yu.N. Obukhov, *An assessment of Evans' unified field theory II*, both in Foundations of Physics, to be published (2007). See also http://arXiv.org/abs/physics/0703116 and http://arXiv.org/abs/physics/0703117