FROM QUANTUM PROBABILITIES TO CLASSICAL FACTS

Ph. Blanchard
Faculty of Physics and BiBoS
University of Bielefeld
D-33615 Bielefeld

A. Jadczyk
Institute of Theoretical Physics
University of Wroclaw
Pl. Maxa Borna 9
PL-50204 Wroclaw

a Gianfausto Dell'Antonio con stima ed amicizia

Abstract

Model interactions between classical and quantum systems are briefly reviewed. These include: general measurement - like couplings, Stern-Gerlach experiment, model of a counter, quantum Zeno effect, piecewise deterministic Markov processes and meaning of the wave function.

1. Introduction

Quantum theory seems to be in conflict with so much of the real world but agrees so perfectly well with observations and describes and computes so much from the behaviour of the solids, the colour of the stars up to the structure and the function of DNA that almost all physicists accepted the fascinating but hard to follow interpretation of its mathematical structure proposed by N. Bohr [1]. As emphasized by F. Rohrlich [2] "Quantum Theory is the greatest conceptual revolution of our century and probably the greatest that mankind had ever experienced ...". In classical statistical mechanics we have to do with ontic determinism but epistemic indeterminism. In Quantum Mechanics the probabilistic description is the fundamental description and no deeper level exists i.e. we have ontic indeterminism (what events are possible and how possible each of them is). The quantum state contains a lot of potentialities and the collapse creates Heisenberg's transition from the possible to the actual. Orthodox Quantum Mechanics considers two types of incompatible time evolution U and R, R denoting the reduction of the quantum state. U is linear, deterministic, local, continuous and time reversal invariant. On the other hand R is probabilistic, non-linear, discontinuous, anticausal (making events consequences of their observations and determines a stochastic change of reality in time). For a fundamental physical theory this situation is not very satisfactory but it works for all practical purposes!

In recent papers [3], [4] we propose a mathematically consistent model describing the interaction between classical and quantum systems. It provides an answer to the question of how and why quantum phenomena become real as a result of interaction between quantum and classical domains. Our results show that a simple dissipative time evolution can allow a dynamical exchange of information between classical and quantum levels of Nature. Indeterminism is an implicit part of classical physics. Irreversible laws are fundamental and reversibility is an approximation. R. Haag formulated the same thesis as "... once one accepts indetermination there is no reason against including irreversibility as part of the fundamental laws of Nature" [5].

With a properly chosen initial state the quantum probabilities are exactly mirrored by the state of the classical system and moreover the state of the quantum subsystem converges as $t \to +\infty$ to a limit in agreement with von Neumann-Lüders standard quantum mechanical measurement projection postulate R. In our model the quantum system Σ_q is coupled to a classical recording device Σ_c which will respond to its actual state. Σ_q should affect Σ_c, which should therefore be treated dynamically. We thus give a minimal mathematical semantics to describe the measurement process in Quantum Mechanics. For this reason the simplest model that we proposed can be seen as the elementary building block used by Nature in the constant communications that take place between the quantum and classical levels. We propose to consider the total system $\Sigma_{tot} = \Sigma_q \otimes \Sigma_c$ and the behaviour associated to the total algebra of observables $\mathcal{Q}_{tot} = \mathcal{Q}_q \otimes \mathcal{Q}_c = \mathcal{C}(X_c) \otimes \mathcal{L}(\mathcal{H}_q)$, where X_c is the classical phase space and $\mathcal{H}_q$ the Hilbert space associated to Σ_q, is now taken as the fundamental reality with pure quantum behaviour as an approximation valid in the cases when recording effects can be neglected. In $\mathcal{Q}_{tot}$ we can describe irreversible changes occuring in the physical world, like the blackening photographic emulsion, as well as idealized reversible pure quantum and pure classical processes. We extend the model of Quantum Theory in such a way that the successful features of the existing theory are retained but the transitions between equilibria in the sense of recording effects is permitted. In Section 2 we will briefly describe the mathematical and physical ingredients of the model and discuss the measurement process in this framework.

The range of applications of the model is rather wide as will be shown in Section 3 with a discussion of Zeno effect. To the Liouville equation describing the time evolution of statistical states of Σ_{tot} we will be in position to associate a piecewise deterministic process taking values in the set of pure states of Σ_{tot}. Knowing this process one can answer all kinds of questions about time correlations of the events as well as simulate numerically the possible histories of individual quantum-classical systems. Let us emphasize that nothing more can be expected from a theory without introducing some explicit dynamics of hidden variables. What we achieved is the maximum of what can be achieved, which is more than orthodox interpretation gives. There are also no paradoxes; we cannot predict, but we can simulate the observations of individual systems. Moreover, we will briefly comment on the meaning of the wave function. Section 4 deals with some other applications and concluding remarks.

2. Measurement-like processes

For a long time the theory of measurement in quantum mechanics, elaborated by Bohr, Heisenberg und von Neumann in the 1930s has been considered as an esoteric subject of little relevance for real physics. But in the 1980s the technology has made possible to transform "Gedankenexperimente" of the 1930s into real experiments. This progress implies that the measurement process in quantum theory is now a central tool for physicists testing experimentally by high-sensitivity measuring devices the more esoteric aspects of Quantum Theory.

Quantum mechanical measurement brings together a macroscopic and a quantum system.

2.1 Interacting classical and quantum systems

Let us briefly describe the mathematical framework we will use. A good deal more can be said and we refer the reader to [3,4]. Our aim is to describe a non-trivial interaction between a quantum system $\sum_q$ in interaction with a classical system $\sum_c$. To the quantum system there corresponds a Hilbert space $\mathcal{H}_q$. In $\mathcal{H}_q$ we consider a family of orthonormal projectors $e_i = e_i^* = e_i^2$, $(i = 1, ..., n)$, $\sum_{i=1}^n e_i = 1$, associated to an observable $A = \sum_{i=1}^n \lambda_i e_i$ of the quantum mechanical system. The classical system is supposed to have m distinct pure states, and it is convenient to take $m \geq n$. The algebra $\mathcal{A}_c$ of classical observables is in this case nothing else as $\mathcal{A}_c = \mathbf{C}^m$. The set of classical states coincides with the space of probability measures. Using the notation $X_c = \{s_0, ..., s_{m-1})$, a classical state is therefore an m-tuple $p = (p_0, ..., p_{m-1}), p_\alpha \geq 0, \ \sum_{\alpha=0}^{m-1} \ p_\alpha = 1$. The state s_0 plays in some cases a distinguished role and can be viewed as the neutral initial state of a counter. The algebra of observables of the total system $\mathcal{A}_{tot}$ is given by

$$\mathcal{A}_{tot} = \mathcal{A}_c \otimes L(\mathcal{H}_q) = \mathbf{C}^m \otimes L(\mathcal{H}_q) = \bigoplus_{\alpha=0}^{m-1} \ L(\mathcal{H}_q), \tag{1}$$

and it is convenient to realize $\mathcal{A}_{tot}$ as an algebra of operators on an auxiliary Hilbert space $\mathcal{H}_{tot} = \mathcal{H}_q \otimes \mathbf{C}^m = \bigoplus_{\alpha=0}^{m-1} \ \mathcal{H}_q$. $\mathcal{A}_{tot}$ is then isomorphic to the algebra of block diagonal $m \times m$ matrices $A = diag(a_0, a_1, ..., a_{m-1})$ with $a_\alpha \in L(\mathcal{H}_q)$. States on $\mathcal{A}_{tot}$ are represented by block diagonal matrices

$$\rho = diag(\rho_0, \rho_1, ..., \rho_{m-1}) \tag{2}$$

where the ρ_α are positive trace class operators in $L(\mathcal{H}_q)$ satisfying moreover $\sum_\alpha \ Tr(\rho_\alpha) = 1$. By taking partial traces each state ρ projects on a 'quantum state' $\pi_q(\rho)$ and a 'classical state' $\pi_c(\rho)$ given respectively by

$$\pi_q(\rho) = \sum_\alpha \rho_\alpha, \tag{3}$$

$$\pi_c(\rho) = (Tr\rho_0, Tr\rho_1, ..., Tr\rho_{m-1}). \tag{4}$$

The time evolution of the total system is given by a semi group $\alpha^t = e^{tL}$ of positive maps[1] of $\mathcal{A}_{tot}$– preserving hermiticity, identity and positivity – with L of the form

$$L(A) = i[H, A] + \sum_{i=1}^{n} (V_i^* A V_i - \frac{1}{2}\{V_i^* V_i, A\}). \tag{5}$$

The V_i can be arbitrary linear operators in $L(\mathcal{H}_{tot})$ such that $\sum V_i^* V_i \in \mathcal{A}_{tot}$ and $\sum V_i^* A V_i \in \mathcal{A}_{tot}$ whenever $A \in \mathcal{A}_{tot}$, H is an arbitrary block-diagonal self adjoint operator $H = diag(H_\alpha)$ in $\mathcal{H}_{tot}$ and $\{,\}$ denotes anticommutator i.e.

$$\{A, B\} \equiv AB + BA. \tag{6}$$

In order to couple the given quantum observable $A = \sum_{i=1}^n \lambda_i e_i$ to the classical system, the V_i are chosen as tensor products $V_i = \sqrt{\kappa} e_i \otimes \phi_i$, where ϕ_i act as transformations on classical (pure) states. Denoting $\rho(t) = \alpha_t(\rho(0))$, the time evolution of the states is given by the dual Liouville equation

$$\dot{\rho}(t) = -i[H, \rho(t)] + \sum_{i=1}^{n} (V_i \rho(t) V_i^* - \frac{1}{2}\{V_i^* V_i, \rho(t)\}), \tag{7}$$

[1] In fact, the maps we use happen to be also completely positive.

where in general H and the V_i can explicitly depend on time.
Remarks:
1) It is possible to generalize this framework for the case where the quantum mechanical observable A we consider has a continuous spectrum (as for instance in a measurement of the position) with $A = \int_{\mathbf{R}} \lambda dE(\lambda)$. See [6,7] for more details.
2) Since the center of the total algebra $\mathcal{A}_{tot}$ is invariant under any automorphic unitary time evolution, the Hamiltonian part H of the Liouville operator is not directly involved in the process of transfer of information from the quantum subsystem to the classical one. Only the dissipative part can achieve such a transfer in a finite time.

2.2 The quantum mechanical measurement process

In [3] we propose a simple, purely dissipative Liouville operator (i.e. we put $H = 0$) that describes an interaction of $\sum_q$ and $\sum_c$, for which $m = n + 1$ and $V_i = e_i \otimes \phi_i$, where ϕ_i is the flip transformation of X_c transposing the neutral state s_0 with s_i. We show that the Liouville equation can be solved explicitly for any initial state $\rho(0)$ of the total system. Assume now that we are able to prepare at time $t = 0$ the initial state of the total system Σ_{tot} as an uncorrelated product state $\rho(0) = w \otimes P^\epsilon(0), P^\epsilon(0) = (p_0^\epsilon, p_1^\epsilon, ..., p_n^\epsilon)$ as initial state of the classical system parametrized by $\epsilon, 0 \leq \epsilon \leq 1$:

$$p_0^\epsilon = 1 - \frac{n\epsilon}{n+1}, \tag{8}$$

$$p_1^\epsilon = \frac{\epsilon}{n+1}. \tag{9}$$

In other words for $\epsilon = 0$ the classical system starts from the pure state $P(0) = (1, 0, ..., 0)$ while for $\epsilon = 1$ it starts from the state $P'(0) = (\frac{1}{n+1}, \frac{1}{n+1}, ..., \frac{1}{n+1})$ of maximal entropy. Computing $p_i(t) = Tr(\rho_i(t))$ and then the normalized distribution

$$\tilde{p}_i(t) = \frac{p_i(t)}{\sum_{r=1}^{n} p_r(t)} \tag{10}$$

with $\rho(t) = (\rho_0(t), \rho_1(t), ..., \rho_n(t))$ the state of the total system we get:

$$\tilde{p}_i(t) = q_i + \frac{\epsilon(1 - nq_i)}{\epsilon n + \frac{(1-\epsilon)(n+1)}{2}(1 - e^{-2\kappa t})}, \tag{11}$$

where we introduced the notation

$$q_i = Tr(e_i w), \tag{12}$$

for the initial quantum probabilities to be measured. For $\epsilon = 0$ we have $\tilde{p}_i(t) = q_i$ for all $t > 0$, which means that the quantum probabilities are exactly, and immediately after switching on of the interaction, mirrored by the state of the classical system. For $\epsilon = 1$ we get $\tilde{p}_i(t) = 1/n$. The projected classical state is still the state of maximal entropy and in this case we get no information at all about the quantum state by recording the time evolution of the classical one. In the intermediate regime, for $0 < \epsilon < 1$, it is easy to show that $|\tilde{p}_i(t) - q_i|$ decreases at least as $2\epsilon(1 + e^{-2\kappa t})$ with $\epsilon \to 0$ and $t \to +\infty$. For $\epsilon = 0$, that is when the measurement is exact, we get for the partial quantum state

$$\pi_q(\rho(t)) = \sum_i e_i w e_i + e^{-\kappa t}(w - \sum_i e_i w e_i),$$

so that

$$\pi_q(\rho(\infty)) = \sum_i e_i w e_i, \tag{13}$$

which means that the partial state of the quantum subsystem $\pi_q(\rho(t))$ tends for $\kappa t \gg +\infty$ to a limit which coincides with the standard von Neumann-Lüders quantum measurement projection postulate.

Remark:

The normalized distribution $\tilde{p}_i(t)$ is nothing else as the read off from the outputs $s_1 ... s_n$ of the classical system $\sum_c$.

2.3 Efficiency versus accuracy by measurement

Let us consider the case where

$$V_i = \sqrt{\kappa} e_i \otimes f_i, \tag{14}$$

f_i being the transformation of X_c mapping s_0 into s_i. In the Liouville equation we consider also an Hamiltonian part. We find for the Liouville equation:

$$\dot{\rho}_0 = -i[H, \rho_0] - \kappa \rho_0, \tag{15}$$

$$\dot{\rho}_i = -i[H, \rho_i] + \kappa e_i \rho_0 e_i, \tag{16}$$

where we allow for time dependence i.e. $H = H(t), e_i = e_i(t)$. Setting $r_0(t) = Tr(\rho_0(t))$, $r_i(t) = Tr(\rho_i(t))$, and assuming that the initial state is of the form $\rho = (\rho_0, 0, ..., 0)$ we conclude that $\dot{r}_0 = -\kappa r_0$ and thus $r_0(t) = e^{-\kappa t}$ which implies that

$$\sum_{i=1}^{n} r_i(t) = 1 - e^{-\kappa t}, \tag{17}$$

from which it follows that a 50 % efficiency requires $\log 2/\kappa$ time of recording. It is easy to compute $r_i(t)$ and

$$\tilde{p}_i(t) = \frac{r_i(t)}{\sum_{j=1}^{n} r_j(t)}$$

for small t. We get

$$\tilde{p}_i(t) = q_i + \frac{\kappa^2 t^2}{2}\frac{1}{\kappa}\langle\frac{de_i}{dt}\rangle_{\rho_0} + o(t^2), \tag{18}$$

where

$$\frac{de_i}{dt} \doteq \frac{\partial e_i}{\partial t} + i[H, e_i]. \tag{19}$$

Efficiency requires $\kappa t >> 1$ while accuracy is achieved if $(\kappa t)^2 << \frac{\kappa}{\langle \dot{e}_i\rangle_{\rho_0}}$. To monitor effectively and accurately non stationary processes we must therefore take $t << \langle \dot{e}_i\rangle_{\rho_0}$ and $1/\kappa << t$. If however H and e_i does not depend on time, then if either $\rho_0(0)$ or e_i commutes with H, we get $\tilde{p}_i(t) = q_i$ exactly and instantly.

When discussing measurement problem we may also ask the question about the value of disturbance of the initial quantum state owing to the measurement. Here we may look at the Eqns. (15-16) and compare them to the undisturbed evolution ($\kappa = 0$) . Denoting by $\hat{\rho} = \sum_\alpha \rho_\alpha$ the partial state of the quantum system, we get for the difference of rates the two evolutions

$$\delta\dot{\hat{\rho}} = \kappa(\sum_i e_i\rho_0 e_i - \rho_0).$$

But for a generic initial state ρ_0, the quantity $\sum_i e_i\rho_0 e_i - \rho_0$ can have trace norm of order 1. It follows that the disturbance of the state during a short coupling time t can be of the order κt. Comparing this to the discussion of efficiency, we conclude that efficiency of the counters and their nondemolition property can be considered in the model that we have discussed as being complementary to each other.

2.4 Stern Gerlach experiment

In the spirit of A. Böhm (cf. Ref. [8, Ch. XIII]) we model a Stern-Gerlach device by a pure spin 1/2 particle interacting with a spinless atom. Assuming that the magnetic field is linear in z the interaction Hamiltonian can be written

$$H_{int} = 2\mu_B B z \sigma_3. \tag{20}$$

Writing

$$\sigma_3 = \frac{|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|}{2},$$

H_{int} is now given by

$$H_{int} = \mu_B B(|\uparrow\rangle\langle\uparrow|z + |\downarrow\rangle\langle\downarrow|(-z)). \tag{21}$$

Supposing now that the atom can be directly observed we can replace it for all practical purposes by a 3-state classical device (s_0, s_+, s_-). The coupling is then modelled by

$$\sqrt{\kappa}(p flip(0 \to +) + (1-p) flip(0 \to -)) \tag{22}$$

where $p = \frac{1}{2}(\sigma_0 + \underline{n\sigma})$ is the projection onto the spin component to be measured. We are now in position to approximate Stern-Gerlach experiment by our 3-state model. For more details see also [6].

2.5 Model of a counter

We consider as in [9] a one-dimensional ultra-relativistic quantum mechanical particle. This nice model can be solved exactly and provides a clear understanding of the physical phenomena at work. The counter sensitivity is described by an operator valued function $f(t)$ and the quantum system $\sum_q$ with $\mathcal{H}_q = L^2(\mathbf{R}, dx)$ is coupled to a 2-state classical system. The Liouville equation for the state of the total system is

$$\dot{\rho} = -i[H, \rho] + V\rho V^* - \frac{1}{2}\{V^*V, \rho\}, \tag{23}$$

with

$$V = f \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & f \\ 0 & 0 \end{pmatrix} \tag{24}$$

Now explicitly we obtain

$$\dot{\rho}_0 = -i[H, \rho_0] - \frac{1}{2}\{f^*f, \rho_0\}, \tag{25}$$

$$\dot{\rho}_1 = -i[H, \rho_1] + f\rho_0 f^*. \tag{26}$$

Taking $H = \frac{1}{i}\frac{d}{dx}$ and $f = f^* = f(x,t)$ we obtain for the counting rate $\dot{\rho}_1(t)$ in a free evolving state:

$$\dot{p}_1(t) = \int_{\mathbf{R}} |\Psi(x-t)|^2 f^2(x,t) e^{-\int_0^t f^2(x+s-t,s)ds} dx. \tag{27}$$

Assume now that we have to do with a point article i.e.

$$|\Psi(x)|^2 = \delta(x - x_0)$$

we obtain in this idealized case

$$\dot{p}_1(t) = f^2(x_0 + t, t) e^{-\int_0^t f^2(x_0+s,s)ds} \tag{28}$$

which expresses the fact that the counting rate depends on how long the "detector" was already in contact with the particle. It is possible to generalize the results obtained to the case of a nonrelativistic quantum mechanical particle for which $H = -\frac{d^2}{dx^2}$. In this situation the model would remain solvable within reasonable approximations. For more details see [6]. Moreover for the ultrarelativistic dynamics the multidetector case can be solved completely.

3. Quantum Zeno Effect

For a rapid sequence of measurements made at times $k\tau$ which are multiples of a small unit time τ, a quantum system will not change at all. It seems as if time has stood still for it. This effect known as Zeno effect can be understood in terms of quantum mechanical perturbation theory. Indeed it gives for small times a transition probability per unit of time

$$p_1(\tau) = 1 - | < \psi_0, e^{i\tau H}\psi_0 > |^2 \simeq (\Delta H)^2 \tau^2$$

with

$$(\Delta H)^2 = \langle \psi_0, H^2 \psi_0 \rangle - | \langle \psi_0, H\psi_0 \rangle |^2 \geq 0$$

and therefore $p_1(\tau)$ vanishes quadratically in the limit $\tau \to 0$. For larger values of time the expected constant rate is formal. This result must be explained if Quantum Theory is to continue to make sense. The most obvious explanation is that any actual measurement requires a time T and the paradox is eliminated if it can be shown that $T > \tau$.

This effect was formulated by Turing 1940 and called 1977 Quantum Zeno effect by Misra and Sudarshan [10]. In recent years there has been considerable discussion of the quantum Zeno process, effect and paradox. See for example [11, 12, 13, 14]. Moreover it has been claimed that experiments can demonstrate the effect [15, 16, 17].

3.1 Quantum Zeno Effect revisited

Using our model of a continuous measurement we can easily discuss this effect for a quantum spin 1/2 system coupled to a 2-state classical system [18]. We consider only one orthogonal projector $e = e^* = e^2$ on the two-dimensional Hilbert space $\mathcal{H}_q = \mathbf{C}^2$.

To define the dynamics we choose the coupling operator V in the following way:

$$V = \sqrt{\kappa} \begin{pmatrix} 0, & e \\ e, & 0 \end{pmatrix}. \tag{29}$$

The Liouville equation (7) for the density matrix $\rho = diag(\rho_0, \rho_1)$ of the total system reads now

$$\begin{aligned} \dot{\rho}_0 &= -i[H, \rho_0] + \kappa(e\rho_1 e - \frac{1}{2}\{e, \rho_0\}), \\ \dot{\rho}_1 &= -i[H, \rho_1] + \kappa(e\rho_0 e - \frac{1}{2}\{e, \rho_1\}). \end{aligned} \tag{30}$$

For this particularly simple coupling the effective quantum state $\hat{\rho} = \pi_q(\rho) = \rho_0 + \rho_1$ evolves independently of the state of the classical system, expressing the fact that here we have only transport of information from the quantum system to the classical one. We have:

$$\dot{\hat{\rho}} = -i[H, \hat{\rho}] + \kappa(e\hat{\rho}e - \frac{1}{2}\{e, \hat{\rho}\}). \tag{31}$$

For the discussion of the quantum Zeno effect we specialize:

$$\begin{aligned} H &= \frac{\omega}{2}\sigma_3, \\ e &= \frac{1}{2}(\sigma_0 + \sigma_1), \end{aligned} \tag{32}$$

σ_i being the Pauli matrices, $i = 0, 1, 2, 3$.

We start with the quantum system being initially in the eigenstate of σ_1, and repeatedly (with "frequency" κ) check if the system is still in this state, each "yes" causing a flip in the coupled classical device - which we can continuously observe.

The evolution equation for $\hat{\rho}$, with the initial condition $\hat{\rho}(t=0) = e$, can be exactly solved with the result:

$$\hat{\rho}(t) = \frac{1}{2}(\sigma_0 + x(t)\sigma_1 + y(t)\sigma_2), \tag{32}$$

where $x(t), y(t)$ are given by

$$\begin{aligned} x(t) &= \exp\left(\frac{-\kappa t}{4}\right)\left(\cosh\left(\frac{\kappa_\omega t}{4}\right) + \frac{\kappa}{\kappa_\omega}\sinh\left(\frac{\kappa_\omega t}{4}\right)\right), \\ y(t) &= \frac{4\omega}{\kappa_\omega}\exp\left(\frac{-\kappa t}{4}\right)\sinh\left(\frac{\kappa_\omega t}{4}\right), \end{aligned} \tag{33}$$

where $\kappa_\omega = \sqrt{\kappa^2 - 16\omega^2}$. Let us introduce the dimensionless characteristic coefficient $\alpha = \frac{\kappa}{4\omega}$. For $\alpha > 1$ oscillations are damped completely, and then the distance travelled by the quantum state during the interaction becomes inversely proportional to the square root of α. The natural distance in the state space is the geodesic Bures-Uhlmann distance $d_\frown$, which is the geodesic distance for the Riemannian metric - given in our case by $ds^2 = g_{ij}dx^i dx^j$, with $g_{ij}(\mathbf{v}) = (\delta_{ij} + v_i v_j/(1-\mathbf{v}^2))$. For density matrices $v = (\sigma_0 + \mathbf{v}\cdot\sigma)/2$ and $\omega = (\sigma_0 + \mathbf{w}\cdot\sigma)/2$ we obtain

$$d(v \frown w) = \frac{1}{2}\arccos(\mathbf{v}\cdot\mathbf{w} +^4 \sqrt{1-\mathbf{v}^2}^4\sqrt{1-\mathbf{w}^2}). \tag{34}$$

In particular, if one of the states, say v, is pure, then $\mathbf{v}^2 = 1$ and $d(v \frown w)$ is simply given by

$$d(v \frown w) = \frac{1}{2}\arccos(\mathbf{v}\cdot\mathbf{w}). \tag{35}$$

For $v = \hat{\rho}(t), w = e = (\sigma_0 + \sigma_1)/2$, as in the Zeno model, we obtain

$$d(\hat{\rho}(t) \frown e) = \frac{1}{2}\arccos(x(t)). \tag{36}$$

Notice that e, being a pure state, is on the boundary of the state space, and the $d_\frown$-distance from e depends only on one of the two relevant coordinates x, y - contrary to the

Frobenius distance $T_r((v-w)^2)$, which would involve both coordinates. Assuming now $\alpha \gg 1$ and $\kappa t \gg 1$, we get for $x(t), y(t)$ in (33) asymptotic formulae:

$$\begin{aligned} x(t) &\asymp 1 - \frac{2\omega^2 t}{\kappa} + \dots \\ y(t) &\asymp \frac{\omega}{2\kappa} + \dots \end{aligned} \tag{37}$$

Thus the distance reached by state is in this asymptotic region given by

$$d \asymp \omega \sqrt{\frac{t}{\kappa}} \tag{38}$$

3.2 Quantum Zeno effect via piecewise deterministic processes

Our objective in this section is to give a stochastic description of the continuous measurement Zeno effect by using piecewise deterministic processes. A noteworthy reference for this subject is [19]. An observable A of the total system is a pair $(A_\alpha)_{\alpha=0,1}$ of operators in $\mathcal{H}_q$. Every observable A determines a function f_A on pure states of the total system (or a pair of functions f_A^α on pure states of Σ_q) by

$$f_A(\varphi, s_\alpha) \equiv f_A^\alpha(\varphi) \equiv \langle \varphi, A_\alpha \varphi \rangle. \tag{38}$$

We shall exhibit a Markov semigroup generator, M, acting on functions $f(\varphi, s)$ which gives the evolution equations (30) in their dual form:

$$\dot{A}_\alpha = i[H, A_\alpha] + \kappa(eA_\alpha e - \frac{1}{2}\{e, A_{\alpha'}\}) \tag{40}$$

where we use the notation $\alpha' = \alpha + 1 \bmod 2$. Thus we want to have

$$\begin{aligned}(Mf_A)(\varphi, s_\alpha) &= \frac{d}{dt} f_{A(t)}(\varphi, s_\alpha) \mid_{t=0} \\ &= \frac{d}{dt}\langle \varphi, A_\alpha(t)\varphi\rangle \mid_{t=0} = \langle \varphi, \dot{A}_\alpha \varphi \rangle.\end{aligned} \tag{41}$$

To this end we have to rewrite the RHS of (41)

$$\langle \varphi, \dot{A}_\alpha \varphi\rangle = \langle \varphi, i[H, A_\alpha]\varphi\rangle + \kappa\langle \varphi, eA_\alpha\varphi\rangle - \frac{\kappa}{2}\langle \varphi, \{e, A_{\alpha'}\}\varphi\rangle \tag{42}$$

in terms of functions $f_A^\alpha(\varphi) = f_A(\varphi, s_\alpha)$. Note that the first term on the RHS of (42) is equal to

$$\langle \varphi, i[H, A_\alpha]\varphi\rangle = \frac{d}{dt} f_A^\alpha(e^{-iHt}\varphi) \mid_{t=0} . \tag{43}$$

By introducing the vector field X_H on the unit ball of $\mathcal{H}_q$ defined by the Hamiltonian evolution:

$$(X_H f)(\varphi) \doteq \frac{d}{dt} f(e^{-iHt}\varphi) \mid_{t=0} \tag{44}$$

we observe that

$$\langle \varphi, i[H, A_\alpha]\varphi\rangle = (X_H f_A^\alpha)(\varphi). \tag{45}$$

The next step is to observe that

$$-\frac{\kappa}{2}\langle \varphi, \{e, A_\alpha\}\varphi\rangle = \frac{d}{dt}\langle e^{-\frac{\kappa t}{2}e}\varphi, A_\alpha e^{-\frac{\kappa t}{2}e}\varphi\rangle \mid_{t=0}, \tag{46}$$

which will give rise to two terms as follows. Let us introduce another vector field $X_{\mathcal{D}}$ on the unit ball of $\mathcal{H}_q$ by

$$(X_{\mathcal{D}} f)(\varphi) \doteq \frac{d}{dt} f\left(\frac{e^{-\frac{\kappa t}{2}e}\varphi}{\| e^{-\frac{\kappa t}{2}e}\varphi \|}\right) \mid_{t=0} . \tag{47}$$

We are now able to rewrite (46) as

$$-\frac{\kappa}{2}\langle\varphi,\{e,A_\alpha\}\varphi\rangle = (X_{\mathcal{D}} f_A^\alpha)(\varphi) - \lambda(\varphi) f_A^\alpha(\varphi), \tag{48}$$

where we introduced the function $\lambda(\varphi)$

$$\lambda(\varphi) = \kappa \parallel e\varphi \parallel^2 . \tag{49}$$

We can express now the middle term of (42) as

$$\kappa\langle\varphi, eA_{\alpha'} e\varphi\rangle = \lambda(\varphi) f_A^{\alpha'}\left(\frac{e\varphi}{\parallel e\varphi \parallel}\right).$$

It follows that

$$\langle\varphi, \dot{A}_\alpha\varphi\rangle = (X_H + X_{\mathcal{D}}) f_A^\alpha(\varphi) + \lambda(\varphi) A_{\alpha'}\left(\frac{e\varphi}{\parallel e\varphi \parallel}\right) - \lambda(\varphi) A_\alpha(\varphi). \tag{50}$$

We finally introduce the matrix valued measure

$$Q(d\psi,\varphi) = (Q_\alpha^\beta(d\psi,\varphi)) = \begin{pmatrix} 0, & \delta_{\frac{\varphi}{\parallel e\varphi \parallel}} \\ \delta_{\frac{\varphi}{\parallel e\varphi \parallel}}, & 0 \end{pmatrix}, \tag{51}$$

then

$$\begin{aligned}\langle\varphi, \dot{A}_\alpha\varphi\rangle &= (X_H + X_{\mathcal{D}}) f_A^\alpha(\varphi) \\ &\quad + \lambda(\varphi)\sum_\beta \int (f_A^\beta(\psi) - f_A^\alpha(\varphi)) Q_\beta^\alpha(d\psi,\varphi).\end{aligned} \tag{52}$$

It shows that the evolution equation (40) follows from a Markov semigroup of a piecewise deterministic process with generator

$$\begin{aligned}(M,f)(\varphi,s_\alpha) &= [(X_H + X_{\mathcal{D}})f](\varphi,s_\alpha) \\ &\quad + \lambda(\varphi)\sum_\beta \int [f(\psi,\beta) - f(\varphi,\alpha)] Q_\beta^\alpha(d\psi,\varphi).\end{aligned} \tag{53}$$

To this process, as in [19], we can associate a jump process which is not a Markov process and a Feynman-Kac formula can be used to calculate the expectations of functionals. We refer to [20] for technicalities. The information contained in the Liouville equation (39) is therefore not the maximal available one. Knowing the piecewise deterministic Markov process associated to (53) we want to emphasize that we can answer all kinds of questions about time correlations of events and also simulate the random behavior of the classical system Σ_c coupled to the quantum system Σ_q. Let T_t be the one parameter semigroup of non-linear transformations of rays in $\mathbf{C}^2$ given by

$$T_t\phi = \frac{\phi(t)}{\parallel \phi(t) \parallel} \tag{54}$$

where

$$\phi(t) = e^{-iHt - \frac{k}{2}et}\phi \tag{55}$$

Let us now suppose that a $t = 0$ the quantum system Σ_q is in the pure state φ and the classical system in the state s_α. Then φ starts to evolve according to the deterministic non unitary (and non-linear) time evaluation $T_t\varphi$ until jump occurs at time $t_1 > 0$. The random jump-time t_1 is governed by an inhomogeneous Poisson process with rate function

$$\lambda(t) = \kappa \parallel eT_t\varphi \parallel^2 . \tag{56}$$

The classical system switches from s_α to $s_{\alpha'}$, while $T_{t_1}\varphi$ jumps to $eT_{t_1}\varphi / \parallel eT_{t_1}\varphi \parallel$ with probability 1, and the process starts again. If the initial state φ is an eigenstate of e with eigenvalue one

$$e\varphi = \varphi \tag{57}$$

as it is in our Zeno model, and for large values of the coupling constant κ, the intensity λ is nearly constant and equal to κ. Thus $1/\kappa$ can be interpreted as the mean time between succesive jumps. Strong coupling between the classical and the quantum system, which is necessary for the occuring of the Zeno effect, manifests itself by a high frequency of jumps. Notice that the distribution function of the jump time is given by

$$p[t_1 > t] = \exp(-\int_0^t \lambda(T_s\varphi)ds), \tag{58}$$

and so the probability that the jump will occur in the time interval $(t, t + dt)$ is

$$-\frac{dp[t_1 > t]}{dt} = \parallel eT_t\varphi \parallel^2 \exp\left(-\int_0^t \parallel eT_s\varphi \parallel^2 ds\right). \tag{59}$$

Thus at time instants t when $\parallel eT_t\varphi \parallel = 0$, which would cause problems with the formulae (50) and (51), jumps do not occur. In section 4 we will comment on the meaning of these jumps.

We notice that the conventional analysis of the Zeno effect, as given in the analysis of experiment by Itano (see Refs [15], [16]) is in agreement with our framework.

3.3 Remarks on "meaning of the wave function"

It is tempting to use the Zeno effect for slowing down the time evolution in such a way, that the state of a quantum system Σ_q can be determined by carrying out measurements of sufficiently many observables. This idea, however, would not work, similarly like would not work the proposal of "protective measurements" of Y. Aharonov et al (see [21] [22]). To apply Zeno-type measurements just as to apply a "protective measurement" one would have to know the state beforhand. Also, our discussion in Sec 2.3 suggests that obtaining a reliable knowledge of the quantum state may necessarily lead to a significant, irreversible disturbance of the state. This negative statement does not mean that we have shown that the quantum state cannot be objectively determined. We believe however that dynamical, statistical and information-theoretical aspects of the important problem of obtaining a "*maximal reliable knowledge ;, of the unknown quantum state with a least possible disturbance*" are not yet sufficiently understood.

4. Concluding remarks and comments

One aim of this review was to show how the problem of quantum measurements can be tackled by using consistent models of interactions between classical and quantum systems. We believe that the framework we propose is not only in position of describing theoretically the measurement process but is also able of analyzing correctly recent experiments. We described models providing an answer to the problem of how and when a quantum phenomenon becomes real. The central idea of these models is based on a modification of quantum mechanics by introducing dissipative elements in the basic dynamical equation and on allowing for a nontrivial dynamics of central quantities

The minimal piecewise deterministic process introduced in connection with Zeno effect can be used for computing time characteristic of the interaction and also for numerical simulations of the phenomenon. One may ask "are these jumps real?". Our answer is: yes, they are real - to the extent that they can be such. These jumps do not occur in space and time but they occur in $\mathcal{H}_q$. Let us emphasize that they can be detected all the same by monitoring devices that are placed in space and time. Our formalism is on this respect consistent: indeed we give not only a theory of jumps but also give means of their "experimental" detection. The complete theory of monitoring of quantum systems, including the analysis of disturbance and information transfer is still to be worked out.

Bohr and Heisenberg made avery sharp distinction between the classical and the quantum domains. The borderline seemed to coincide with the division between macroscopic and microscopic. With the Josephson junctions we are obliged to accept the existence of macroscopic quantum mechanical systems. In [4] and [23] we show that our framework is very well adapted to give a description of a "mini" SQUID and to discuss the behaviour of the coupled system consisting of a macroscopic classical system (tank circuit) and a single quantum object (SQUID) [cf. Ref 24]. The same method can be applied as well to other problems where the classical system is expected to respond to averages of some quantum observables, like, for instance, classical gravitational field is expected to have as its source averaged energy-momentum tensor of quantized matter.

Science continually reworks its foundation and even the formalism of Quantum Mechanics, in spite of the fact that it fits Nature like a glove is not immune to change. The problem of quantum mechanical measurement has been solved "in practice". Indeed Quantum Mechanics is used daily and it works. Our aim was to analyze and to understand how and why it works. Our results show that the reduction of the wave function is not an added postulate but a necessary consequence of the time evolution of the total system $\Sigma_t = \Sigma_c \times \Sigma_q$. The classical probabilities we obtain depend on the available knowledge about the system, giving us the best predictions possible from the partial information that we have.

Einstein once wrote to Schrödinger that ”the Heisenberg-Bohr tranquilizing philosophy is so delicately contrived that, for the time being, it provides a gentle pillow for the true ”believer”. The pillow we propose does not aspire to be a miraculous youth elixir for Quantum Theory making it universal and true for ever. But perhaps it will be fatter and firmer and will help to stop the bleeding from some open scars ... .

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