

Barrier traversal times using a phenomenological track formation model*

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Abstract

A phenomenological model for a measurement of “barrier traversal times” for particles is proposed. Two idealized detectors for passage and arrival provide entrance and exit times for the barrier traversal. The averaged traversal time is computed over the ensemble of particles detected twice, before and after the barrier. The “Hartman effect” can still be found when passage detectors that conserve the momentum distribution of the incident packet are used.

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The temporal characterization of quantum mechanical tunnelling traces back to early studies by McColl [1]. More recently a paper by Büttiker and Landauer [2] and interest in the subject from various fields (as varied as nuclear and molecular physics, cosmology or semiconductor physics) have triggered a debate that has frequently dealt with the very foundations of quantum mechanics [3]. The interpretation of the quantum mechanical formalism and the wave-particle duality, the quantization ambiguities, the relation between classical and quantum mechanics, or the quantum “measurement problem” are some of the ingredients of this research. These are all difficult and not completely understood matters so, not surprisingly, answering the question “How long does it take to cross a barrier?”, i.e., defining a quantum traversal time has been controversial. (We shall mainly discuss the general concept of “traversal time” instead of a more restrictive “tunnelling time”. However especial attention will be paid to tunnelling conditions in the calculations.)

In this problem the standard quantization procedures are difficult to apply since only a limited number of classical trajectories cross the selected region and “continuous observation” may be required for a measurement. Even so, many proposals exist that generalize in different formal or operational ways the classical concept of traversal time to the quantum case. The debate on the barrier traversal time is essentially a consequence of different conditionings and criteria, added to the bare original question, that privilege one quantum quantity versus the others. As long as the conditioning is made explicit, to make clear that different versions of the original question are being answered, there is no fundamental conflict among seemingly irreconcilable proposals. (Part of the theoretical work –using path integrals [4] or a projector approach [5]– has been devoted to develop comprehensive formalisms that allow to classify and relate many of the possible characteristic quantities.) However not all aspects have yet been investigated. Only the totality of conditionings or additional specificications exhausts the possible information about the barrier traversal in the temporal domain. Within this spirit we shall investigate here a complementary aspect to those we have previously examined [5-10], and to experiments performed on electromagnetic waves to measure “Larmor times” [11]. The objective of this letter is to examine one *operational definition of barrier traversal time* for particles. By “operational” we mean “related to a specific experiment, possibly a “Gedanken” experiment. We shall model an idealized experimental setup inspired by an elementary “classical” recipe: In order to measure the transit

time through a spatial region the first entrance t_1 and first exit times t_2 are measured and their difference $\tau = t_2 - t_1$ is evaluated *for each particle*. In our case the spatial region includes a potential barrier and only the particles detected before *and* after the barrier will be taken into account. If the experiment is repeated many times τ can be averaged and its statistical properties examined. In a previous publication by Muga, Brouard and Sala [6] a related approach was proposed for the quantum case: An average entrance instant $\langle t \rangle^{\text{in}}$ at a and an average exit instant $\langle t \rangle^{\text{out}}$ at b were defined in terms of incident and outgoing current densities,

$$\langle t \rangle_a^{\text{in}} = \frac{\int_0^{t_c} J(a)tdt}{\int_0^\infty J(a)tdt}, \quad (1)$$

$$\langle t \rangle_b^{\text{out}} = \frac{\int_0^\infty J(b)tdt}{\int_0^\infty J(b)tdt}, \quad (2)$$

with a traversal time $\tau_T \equiv \langle t \rangle_a^{\text{out}} - \langle t \rangle_b^{\text{in}}$ given by the difference between the two averages. Here J is the current density and it is assumed that a is far from the barrier so that the packet passes rightwards through point a before t_c , a time prior to the backwards reflected flow after the collision. τ_T is in principle measurable but it has a clear drawback since it is not the average of transit times for *individual* particles. It is instead the difference between two averages of different nature. This is better understood in classical terms: The average entrance instant $\langle t \rangle^{\text{in}}$ is operationally defined for the ensemble of particles that arrive at the first detector while the exit time is only defined for a smaller set (those that arrive at the final detector). This definition in fact may lead to negative values of τ_T in the classical and quantum cases [12,7]. Classically the average entrance time may be dominated by trajectories that are eventually reflected so that $\langle t \rangle^{\text{in}}$ can be very different from typical entrance times of the trajectories that eventually pass the barrier. In this letter this inconsistency with the classical limit is overcome by restricting the averaging to those particles that are detected before *and* after the barrier. In general this approach implies a “back reaction” of the first detector that modifies the state. We accept this perturbation as a fact and investigate the outcome of the described operational procedure, and the effect of different detectors, in particular of those that minimize the back reaction so that the momentum distribution of the initial packet is preserved.

In general the particle+detector system involves many degrees of freedom and it is rarely modelled accurately. The objective of a phenomenological

model is to retain its essential aspects with the aid of some adjustable set of parameters and in agreement with experiential facts. Our model does not specify the particular features of the detection at a detailed experimental level but we have in mind *particle tracks* similar to the ones produced in a bubble chamber or by means of photographic plates. These tracks are characterized by a discrete set of macroscopic spots (two in our case) originated at certain times (“clicks”) considered as “classical events” that result from the quantum particle passage or arrival. The particle is restricted to one dimensional spatial motion. Specifically the effect of the detector associated with a given spot is simulated according to a track formation model proposed by A. Jadczyk and Ph. Blanchard using two basic elements [13]: An effective one-degree-of-freedom Hamiltonian and a modified projection postulate for the particle state after the first detection.

1 Model description

The initial state of the particle is given by a wave function ψ associated with a preparation procedure. In operational terms, an ensemble of noninteracting particles, represented symbolically as $\{E_0\}$, is sent towards the barrier - one particle at a time- from the left with identical specifications. (In our calculations the initial state at $t = 0$ is a minimum-uncertainty-product Gaussian centered at position $x = 20$, momentum $p = 8$ and spatial variance $9/4$, all quantities in atomic units. The potential barrier is a square barrier with “height” $V_0 = 50$ from $x = 80$ to $x = 80 + d$ and the particle has mass $m = 1$.)

Two particle detectors A and B are located on both sides of a barrier potential, at $x = a$ and $x = b$. The first one is a passage detector that does not destroy the particle. The second one is an arrival detector. The translational degree of freedom of the particle, x , is the only one represented explicitly. A simplifying assumption is that only one of the two detectors is working at a time: When the particle is sent to the barrier only A is active. Detection of the particle at A disconnects this detector and activates the second, B .

1.1 First detector: Probability of detection

It can be proved using multichannel scattering theory techniques that the incident channel amplitude (corresponding to translational motion of the particle and the detector A in its lower state) can be represented by an effective Schrödinger equation with a complex potential [15]. (In “Event Enhanced Quantum Theory” as described in [13] the imaginary part of the potential is deduced rigorously from the Lindblad form of the Liouville equation that describes a coupling of the quantum system with a classical detector.) Here the effective Schrödinger equation is written as

$$H\psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t) + [V(x) + \Lambda(x)]\psi(x, t), \quad (3)$$

where $V(x)$ represents the potential barrier and the complex potential, Λ , is written as

$$\Lambda(x) = -\frac{i}{2} g^2(x; a), \quad (4)$$

with

$$g(x, a) = s e^{-(x-a)^2/2\sigma^2}. \quad (5)$$

The “intensity”, s , and “width”, σ , of the detector are adjustable parameters.

The norm of the incident channel,

$$N(t) = \int_{-\infty}^{\infty} \psi^*(x, t)\psi(x, t) dx, \quad (6)$$

decreases, due to the detector presence, from the initial value $N(0) = 1$. The total absorption $1 - N(\infty)$ is the *efficiency* of the detector. It is not necessarily equal to one so the ensemble of particles detected at A , $\{E_a\}$, is generally smaller than $\{E_0\}$. The normalized probability density for triggering the detector at time t_a is proportional to the absorption rate $-dN/dt|_{t_a}$. Normalizing with respect to the ensemble $\{E_a\}$ it is given by

$$P(t_a|E_a) = \frac{dN(t_a)/dt_a}{\int_0^\infty dt dN(t)/dt}. \quad (7)$$

1.2 Effect of detection on the wave functions

It will be assumed, within the spirit of a simplified phenomenological model, that after each detection (a “click”) the state of the particle can be effectively

represented by a modified wavefunction. The true final states should be determined by a detailed analysis of the interaction between the system and the detector. Instead we shall later assume a physically motivated functional form. The ensemble of detected particles can be represented by a statistical mixture of such states. This is of course reminiscent of Von Neumann's projection postulate. However an important feature of a bubble chamber track is that it does not look like a random walk. This cannot be explained with a naive projection localizing the particle position by means of position eigenstates, since a position eigenstate has equal probability to expand in any direction (erasing the memory of the state previous to the measurement) so there would be no tendency to ionize atoms in the direction of the dominant incident momentum [16]. A modified projection postulate correcting this fact has been derived by Jadczyk and Blanchard. The wave function resulting from a click at time t_a and consistent with track formation has a memory of the previous state and reflects also the detector properties. A simple expression satisfying these two conditions is [13]

$$\psi_{t_a}(x) = \frac{g(x)\psi(x, t_a)}{[\int_{-\infty}^{\infty} g^2(x)|\psi(x, t_a)|^2 dx]^{1/2}}, \quad (8)$$

where $\psi(x, t_a)$ is the wave function evolved with the Schrödinger equation (3).

To determine the effect of the detector we have examined the momentum average and its variance for the ensembles $\{E_0\}$ and $\{E_a\}$. Averages over $\{E_a\}$ require some care since they imply a double average: The first one (represented as Q) is a quantum mechanical average using each wave packet ψ_{t_a} ; the second (D) is an average over the times of detection t_a weighted by $P(t_a|E_a)$,

$$\langle p \rangle_{E_a} = DQp \equiv \int P(t_a|E_a) \langle \psi_{t_a} | \hat{p} | \psi_{t_a} \rangle dt_a. \quad (9)$$

Since there are two types of average different “variances” are possible [17,18]. For the ensemble $\{E_a\}$ the important one is $\Delta_{DQ}^2 \equiv DQ[p^2 - (DQp)^2]$. (This is a variance computed over detected particles regardless of their detection time [17].) The average momentum is conserved well (especially by weak detectors) except for very narrow detector widths. For all detectors used in this work $DQp \approx \langle p \rangle_{E_0}$ better than 0.2%. However the “momentum widths” Δ_{DQ} (square root of variance) may change drastically with respect to the momentum width Δ_p of the original packet. Fig. 1 shows that wider detectors

tend to keep the variance of the original state while narrow detectors give very large variances. Weak detectors (small s) conserve the variance better than strong detectors (large s). In summary, in our model weak and wide detectors are the best as far as conservation of the momentum distribution of the original packet is concerned. They are however not very efficient, for $s = 1$ the absorbed norm goes from 0.05 to 0.6 in the σ -interval of Fig. 1. In comparison the full norm is absorbed for $s = 10$.

1.3 The second (arrival) detector

The second detector is assumed to be a perfect one as described in [8], so that the full transmitted packet is absorbed. It is located at the right edge of the barrier. Let $\{E_b\}$ be the ensemble of particles that produce two clicks at times t_a and t_b and $P(E_b|t_a)$ the transmittance of ψ_{t_a} , i.e., the fraction of the norm of ψ_{t_a} that will be transmitted and therefore detected at B [19]. The probability for being detected at B conditioned to having been detected at A is

$$P(E_b|E_a) = \int P(E_b|t_a)P(t_a|E_a) dt_a. \quad (10)$$

Instead of using an expression similar to (7) the distribution of arrival times t_b for a perfect absorber can be approximated accurately by the (normalized) flux *without* absorber [8]. In particular, for a wave packet $\psi_{t_a}(x; t_a)$, the detection probability density at t_b in B , conditioned to having been detected at t_a in A and restricted to the ensemble $\{E_b\}$, is given by

$$P(t_b|E_b, t_a) = \frac{J_{t_a}(b, t_b)}{\int J_{t_a}(b, t_b) dt_b}, \quad (11)$$

where J_{t_a} is the flux for the state ψ_{t_a} . Using Bayes' rule the joint probability density for detection at t_a in A and t_b at B restricted to the ensemble $\{E_b\}$ is given by

$$P(t_b, t_a|E_b) = \frac{P(t_b|E_b, t_a)P(E_b|t_a)P(t_a|E_a)}{\int P(E_b|t_a)P(t_a|E_a) dt_a}. \quad (12)$$

Finally, the probability distribution of $\tau \equiv t_b - t_a$ is computed, for the ensemble $\{E_b\}$, by integrating over t_b and t_a with the delta function $\delta(t_b - t_a - \tau)$,

$$P(\tau|E_b) = \frac{\int P(t_a + \tau|E_b, t_a)P(E_b|t_a)P(t_a|E_a) dt_a}{\int P(E_b|t_a)P(t_a|E_a) dt_a}. \quad (13)$$

We have calculated average traversal times $\langle \tau \rangle_{E_b} \equiv \int P(\tau|E_b)\tau d\tau$ versus the barrier width d for two different weak detectors at a , both with $s = 1$. One of them, A_1 , is a wide one and conserves well the momentum distribution of $\{E_0\}$. The other one, A_2 , is a narrow detector, and produces a momentum variance which is approximately ten times the initial one. The detector before the barrier is always put far from the barrier ($a = 50$) to compare with the type of Gedanken experiment performed in ref. [6], so that the initial packet may pass through a before interacting significantly with the barrier, and b is located at the right barrier edge. Let τ_1 and τ_2 be the averages corresponding to using the two initial detectors A_1 and A_2 . Figure 2 shows that the Hartman effect, i.e. the fact that the average traversal time does not grow with d (actually it decreases slowly [7]) can still be seen with A_1 until a critical barrier width d_c where the “classical passage” of momenta “above” the barrier starts to dominate [5,9]. When the narrow detector A_2 is used the momentum variance is so large that the transmission is always dominated by fast momenta well above the barrier (We have independently checked this fact by calculating the ratio between transmission due to energies above and below the barrier energy.) so that the behaviour is the one expected classically, i.e., a linear growth of τ_2 with d . Figure 2 also shows τ_T , which is qualitatively very similar to τ_1 . The relation $\tau_T < \tau_1$ is due to the two different ways the average is performed in the initial detector and can be also understood on classical grounds. The right front of the incident packet is dominated by faster momenta and it contributes with more particles to the transmitted ensemble. For computing the later, no distinction is made at a between particles to be transmitted or not. (The effect grows with d until it saturates when the transmission is purely above the barrier.) Note that τ_T could be negative while the times defined in the present work are always, by construction, strictly positive. The “displacement” of the curve τ_a with respect to τ_T (note the difference in the value of the critical barrier) is due to the slight difference in the momentum variances.

In summary a two detector measurement of a particle traversal time has been modelled. Passage detectors conserving the initial wave packet momentum distribution still show the Hartman effect.

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Figure Captions

Figure 1. Square root of the momentum variance after detection, Δ_{DQ} , for $s = 1$ (solid line) and $s = 10$ (dashed line). The dashed-dotted line is the reference value of the momentum variance for the original ensemble $\{E_0\}$.

Figure 2. Average traversal times versus barrier width d evaluated for (a) $s = 1$, $\sigma = 4.5$ (dashed line); (b) $s = 1$, $\sigma = 0.2$ (dashed-dotted line). The average time τ_T is also represented (solid line).



