HOW AND WHEN QUANTUM PHENOMENA BECOME REAL

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Abstract

We discuss recent developments in the foundations of quantum theory with a particular emphasis on description of measurement–like couplings between classical and quantum systems. The SQUID-tank coupling is described in some details, both in terms of the Liouville equation describing statistical ensambles and piecewise deterministic random process describing random behaviour of individual systems.

1 Introduction

Quantum theory is without doubt one of the most successful constructions in the history of theoretical physics and moreover the most powerful theory of physics: Its predictions have been perfectly verified until now again and again. The new conception of Nature proposed by Bohr, Heisenberg and Born was radically different from that of classical physics and several paradoxes have plagued Quantum Physics since its inception. Although the formalism of non relativistic quantum mechanics was constructed in the late 1920's the interpretation of Quantum Theory is still today the most controversial problem in the foundations of physics. The mathematical formalism and the orthodox interpretation of QM are stunningly simple but leave the gate open for alternative interpretations aimed at solving the dilemma lying in the Copenhagen interpretation. "The fact that an adequate presentation of QM has been so long delayed is no doubt caused by the fact that Niels Bohr brainwashed a whole generation of theorists into thinking that the job was done fifty years ago" wrote Murray Gell Mann 1979. It was also John Bell's point of view that "something is rotten" in the state of Denmark and that no formulation of orthodox quantum mechanics was free of fatal flows. This conviction motivated his last publication [1]. As he says "Surely after 62 years we should have an exact formulation of some serious part of quantum mechanics. By "exact" I do not mean of course "exactly true". I only mean that the theory should be fully formulated in mathematical terms, with nothing left to the discretion of the theoretical physicist ...". Orthodox Quantum Mechanics considers two types of incompatible time evolution U and R, U denoting the unitary evolution implied by Schrödinger's equation and R the reduction of the quantum state. U is linear, deterministic, local, continuous and time reversal invariant while R is probabilistic, non-linear, discontinuous and acausal. Two options are possible for completing Quantum Mechanics. According to John Bell [10] "Either the wave functions is not everything or it is not right ...".

In recent papers [2, 3, 4] we propose mathematically consistent models describing the information transfer between classical and quantum systems. The class of models we consider aims at providing an answer to the question of how and why quantum phenomena become real as a result of interaction between quantum and classical domains. Our results show that a simple dissipative time evolution can allow a dynamical exchange of information between classical and quantum levels of Nature. Indeterminism is an implicit part of classical physics and an explicit ingredient of quantum physics. Irreversible laws are fundamental and reversibility is an approximation. R. Haag formulated a similar thesis as "... once one accepts indetermination there is no reason against including irreversibility as part of the fundamental laws of Nature" [5]. According to the standard terminology the joint systems in our models are open. Thus one is tempted to try to understand their behaviour as an effective evolution of subsystems of unitarily evolving larger quantum systems. Although mathematically possible such an enlargement is non-unique. Therefore we prefer to extend the prevailing paradigm and learn as much as possible how to deal directly with open systems and incomplete information.

With a properly chosen initial state the quantum probabilities are exactly mirrored by the state of the classical system and moreover the state of the quantum subsystem converges as $t \to +\infty$ to a limit in agreement with von Neumann-Lüders standard quantum mechanical measurement projection postulate R. In our model the quantum system $\sum_q$ is coupled to a classical recording device $\sum_c$ which will respond to its actual state. $\sum_q$ should affect $\sum_c$, which should therefore be treated dynamically. We thus give a minimal mathematical semantics to describe the measurement process in Quantum Mechanics. For this reason the simplest models that we proposed can be seen as the elementary building blocks used by Nature in the constant communications that take place between the quantum and classical levels. In our framework a quantum mechanical measurement is nothing else as a coupling between a quantum mechanical system $\sum_q$ and a classical system $\sum_c$ via a completely positive semigroup $\alpha_t = e^{tL}$ in such a way that information can be transferred from $\sum_q$ to $\sum_c$. A measurement represents an exchange of information between physical systems and therefore involves entropy production. See [6] where a definition of entropy for non-commutative systems is given, which is based on the concepts of conditional entropy and stationary couplings between $\sum_q$ and $\sum_c$. Moreover Sauvageot and Thouvenot show the equivalence of this definition with the one proposed by Connes, Narnhofer and Thirring [12].

There have been many attempts to explain quantum measurements. For recent reviews see [7, 8, 9]. Our claim is, that whatever the mechanism used to derive models of measurements starting from fundamental interactions is, this mechanism will lead finally to one model of the class we introduced. In fact any realistic situation will reduce to a model of our class since the overwhelming majority of the properties of the counter and the environment are irrelevant from the point of view of statistically predicting the result of a measurement. We propose indeed to consider the total system $\sum_{tot} = \sum_q \otimes \sum_c$, and the behaviour associated to the total algebra of observables $\mathcal{A}_{tot} = \mathcal{A}_q \otimes \mathcal{A}_c = \mathcal{C}(X_c) \otimes \mathcal{L}(\mathcal{H}_q)$, where X_c is the classical phase space and $\mathcal{H}_q$ the Hilbert space associated to $\sum_q$, is now taken as the fundamental reality with pure quantum behaviour as an approximation valid in the cases when recording effects can be neglected. In $\mathcal{A}_{tot}$ we can describe irreversible changes occuring in the physical world, like the blackening photographic emulsion, as well as idealized reversible pure quantum and pure classical processes. We extend the model of Quantum Theory in such a way that the successful features of the exisiting theory are retained but the transitions between equilibria in the sense of recording effects are permitted. In Section 2 we will briefly describe the mathematical and physical ingredients of the simplest model and discuss the measurement process in this framework.

The range of applications of the model is rather wide as will be shown in Section 3 with a discussion of Zeno effect, giving an account of [11]. To the Liouville equation describing the time evolution of statistical states of $\sum_{tot}$ we will be in position to associate a piecewise deterministic process taking values in the set of pure states of $\sum_{tot}$. Knowing this process one can answer all kinds of questions about time correlations of the events as well as simulate numerically the possible histories of individual quantum-classical systems. Let us emphasize that nothing more can be expected from a theory without introducing some explicit dynamics of hidden variables. What we achieved is the maximum of what can be achieved, which is more than orthodox interpretation gives. There are also no paradoxes; we cannot predict individual events (as they are random), but we can simulate the observations of individual systems. Moreover, we will briefly comment on the meaning of the wave function. The purpose of Section 4 is to discuss the coupling between a SQUID and a damped classical oscillating circuit. Section 5 deals there with some concluding remarks.

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2 Communicating classical and quantum systems

Measurements provide the link between theory and experiment and their analysis is therefore one of the most important and sensitive parts of any interpretation.

For a long time the theory of measurements in quantum mechanics, elaborated by Bohr, Heisenberg und von Neumann in the 1930s has been considered as an esoteric subject of little relevance for real physics. But in the 1980s the technology has made possible to transform "Gedankenexperimente" of the 1930s into real experiments. This progress implies that the measurement process in quantum theory is now a central tool for physicists testing experimentally by high-sensitivity measuring devices the more esoteric aspects of Quantum Theory.

Quantum mechanical measurement brings together a macroscopic and a quantum system.

Let us briefly describe the mathematical framework we will use. A good deal more can be said and we refer the reader to [2, 3, 4]. Our aim is to describe a non-trivial transfer of information between a quantum system $\sum_q$ in interaction with a classical system $\sum_c$. To the quantum system there corresponds a Hilbert space $\mathcal{H}_q$. In $\mathcal{H}_q$ we consider a family of orthonormal projectors $e_i = e_i^* = e_i^2,\ (i = 1, ..., n),\ \sum_{i=1}^n e_i = 1$, associated to an observable $A = \sum_{i=1}^n \lambda_i e_i$ of the quantum mechanical system. The classical system is supposed to have m distinct pure states, and it is convenient to take $m \geq n$. The algebra $\mathcal{A}_c$ of classical observables is in this case nothing else as $\mathcal{A}_c = \mathbf{C}^m$. The set of classical states coincides with the space of probability measures. Using the notation $X_c = \{s_0, ..., s_{m-1})$, a classical state is therefore an m-tuple $p = (p_0, ..., p_{m-1}), p_\alpha \geq 0,\ \sum_{\alpha=0}^{m-1} p_\alpha = 1$. The state s_0 plays in some cases a distinguished role and can be viewed as the neutral initial state of a counter. The algebra of observables of the total system $\mathcal{A}_{tot}$ is given by

$$\mathcal{A}_{tot} = \mathcal{A}_c \otimes L(\mathcal{H}_q) = \mathbf{C}^m \otimes L(\mathcal{H}_q) = \bigoplus_{\alpha=0}^{m-1} L(\mathcal{H}_q), \tag{1}$$

and it is convenient to realize $\mathcal{A}_{tot}$ as an algebra of operators on an auxiliary Hilbert space $\mathcal{H}_{tot} = \mathcal{H}_q \otimes \mathbf{C}^m = \bigoplus_{\alpha=0}^{m-1} \mathcal{H}_q$. $\mathcal{A}_{tot}$ is then isomorphic to the algebra of block diagonal $m \times m$ matrices $A = diag(a_0, a_1, ..., a_{m-1})$ with

$a_\alpha \in L(\mathcal{H}_q)$. States on $\mathcal{A}_{tot}$ are represented by block diagonal matrices

$$\rho = diag(\rho_0, \rho_1, ..., \rho_{m-1}) \tag{2}$$

where the ρ_α are positive trace class operators in $L(\mathcal{H}_q)$ satisfying moreover $\sum_\alpha Tr(\rho_\alpha) = 1$. By taking partial traces each state ρ projects on a 'quantum state' $\pi_q(\rho)$ and a 'classical state' $\pi_c(\rho)$ given respectively by

$$\pi_q(\rho) = \sum_\alpha \rho_\alpha, \tag{3}$$

$$\pi_c(\rho) = (Tr\rho_0, Tr\rho_1, ..., Tr\rho_{m-1}). \tag{4}$$

The time evolution of the total system is given by a semi group $\alpha^t = e^{tL}$ of positive maps[1] of $\mathcal{A}_{tot}$– preserving hermiticity, identity and positivity – with L of the form

$$L(A) = i[H, A] + \sum_{i=1}^{n} (V_i^* A V_i - \frac{1}{2}\{V_i^* V_i, A\}). \tag{5}$$

The V_i can be arbitrary linear operators in $L(\mathcal{H}_{tot})$ such that $\sum V_i^* V_i \in \mathcal{A}_{tot}$ and $\sum V_i^* A V_i \in \mathcal{A}_{tot}$ whenever $A \in \mathcal{A}_{tot}$, H is an arbitrary block-diagonal self adjoint operator $H = diag(H_\alpha)$ in $\mathcal{H}_{tot}$ and $\{,\}$ denotes anti-commutator i.e.

$$\{A, B\} \equiv AB + BA. \tag{6}$$

In order to couple the given quantum observable $A = \sum_{i=1}^n \lambda_i e_i$ to the classical system, the V_i are chosen as tensor products $V_i = \sqrt{\kappa} e_i \otimes \phi_i$, where ϕ_i act as transformations on classical (pure) states. Denoting $\rho(t) = \alpha_t(\rho(0))$, the time evolution of the states is given by the dual Liouville equation

$$\dot{\rho}(t) = -i[H, \rho(t)] + \sum_{i=1}^{n} (V_i \rho(t) V_i^* - \frac{1}{2}\{V_i^* V_i, \rho(t)\}), \tag{7}$$

where in general H and the V_i can explicitly depend on time.

Remarks:

1) It is possible to generalize this framework for the case where the quantum mechanical observable A we consider has a continuous spectrum (as for instance in a measurement of the position) with $A = \int_{\mathbf{R}} \lambda dE(\lambda)$. See [13, 14]

[1] In fact, the maps we use happen to be also completely positive.

for more details. It is also straightforward to include simultaneous measurements of noncommuting observables via semi–spectral measures (see [15]).
2) Since the center of the total algebra $\mathcal{A}_{tot}$ is invariant under any automorphic unitary time evolution, the Hamiltonian part H of the Liouville operator is not directly involved in the process of transfer of information from the quantum subsystem to the classical one. Only the dissipative part can achieve such a transfer in a finite time.

In [2] we propose a simple, purely dissipative Liouville operator (i.e. we put $H = 0$) that describes an interaction of $\sum_q$ and $\sum_c$, for which $m = n + 1$ and $V_i = e_i \otimes \phi_i$, where ϕ_i is the flip transformation of X_c transposing the neutral state s_0 with s_i. We show that the Liouville equation can be solved explicitly for any initial state $\rho(0)$ of the total system. Assume now that we are able to prepare at time $t = 0$ the initial state of the total system Σ_{tot} as an uncorrelated product state $\rho(0) = w \otimes P^\epsilon(0), P^\epsilon(0) = (p_0^\epsilon, p_1^\epsilon, ..., p_n^\epsilon)$ as initial state of the classical system parametrized by $\epsilon, 0 \leq \epsilon \leq 1$:

$$p_0^\epsilon = 1 - \frac{n\epsilon}{n+1}, \tag{8}$$

$$p_1^\epsilon = \frac{\epsilon}{n+1}. \tag{9}$$

In other words for $\epsilon = 0$ the classical system starts from the pure state $P(0) = (1, 0, ..., 0)$ while for $\epsilon = 1$ it starts from the state
$P'(0) = (\frac{1}{n+1}, \frac{1}{n+1}, ..., \frac{1}{n+1})$ of maximal entropy. Computing $p_i(t) = Tr(\rho_i(t))$ and then the normalized distribution

$$\tilde{p}_i(t) = \frac{p_i(t)}{\sum_{r=1}^n p_\tau(t)} \tag{10}$$

with $\rho(t) = (\rho_0(t), \rho_1(t), ..., \rho_n(t))$ the state of the total system we get:

$$\tilde{p}_i(t) = q_i + \frac{\epsilon(1 - nq_i)}{\epsilon n + \frac{(1-\epsilon)(n+1)}{2}(1 - e^{-2\kappa t})}, \tag{11}$$

where we introduced the notation

$$q_i = Tr(e_i w), \tag{12}$$

for the initial quantum probabilities to be measured. For $\epsilon = 0$ we have $\tilde{p}_i(t) = q_i$ for all $t > 0$, which means that the quantum probabilities are exactly, and immediately after switching on of the interaction, mirrored by the state of the classical system. For $\epsilon = 1$ we get $\tilde{p}_i(t) = 1/n$. The projected classical state is still the state of maximal entropy and in this case we get no information at all about the quantum state by recording the time evolution of the classical one. In the intermediate regime, for $0 < \epsilon < 1$, it is easy to show that $|\tilde{p}_i(t) - q_i|$ decreases at least as $2\epsilon(1 + e^{-2\kappa t})$ with $\epsilon \to 0$ and $t \to +\infty$. For $\epsilon = 0$, that is when the measurement is exact, we get for the partial quantum state

$$\pi_q(\rho(t)) = \sum_i e_i w e_i + e^{-\kappa t}(w - \sum_i e_i w e_i),$$

so that

$$\pi_q(\rho(\infty)) = \sum_i e_i w e_i, \tag{13}$$

which means that the partial state of the quantum subsystem $\pi_q(\rho(t))$ tends for $\kappa t \to \infty$ to a limit which coincides with the standard von Neumann-Lüders quantum measurement projection postulate.

Remark:

The normalized distribution $\tilde{p}_i(t)$ is nothing else as the read off from the outputs $s_1 ... s_n$ of the classical system $\sum_c$.

To discuss now the interplay between efficiency and accuracy by measurement, let us consider the case where

$$V_i = \sqrt{\kappa} e_i \otimes f_i, \tag{14}$$

f_i being the transformation of X_c mapping s_0 into s_i. In the Liouville equation we consider also an Hamiltonian part. We find for the Liouville equation:

$$\begin{aligned} \dot{\rho}_0 &= -i[H, \rho_0] - \kappa \rho_0, \\ \dot{\rho}_i &= -i[H, \rho_i] + \kappa e_i \rho_0 e_i, \end{aligned} \tag{15}$$

where we allow for time dependence i.e. $H = H(t), e_i = e_i(t)$. Setting $r_0(t) = Tr(\rho_0(t))$, $r_i(t) = Tr(\rho_i(t))$, and assuming that the initial state is of the form $\rho = (\rho_0, 0, ..., 0)$ we conclude that $\dot{r}_0 = -\kappa r_0$ and thus $r_0(t) = e^{-\kappa t}$ which obviously implies that

$$\sum_{i=1}^{n} r_i(t) = 1 - e^{-\kappa t}, \tag{16}$$

from which it follows that a 50 % efficiency requires $\log 2/\kappa$ time of recording. It is easy to compute $r_i(t)$ and

$$\tilde{p}_i(t) = \frac{r_i(t)}{\sum_{j=1}^{n} r_j(t)}$$

for small t. We get

$$\tilde{p}_i(t) = q_i + \frac{\kappa^2 t^2}{2} \frac{1}{\kappa} \langle \frac{de_i}{dt} \rangle_{\rho_0} + o(t^2), \tag{17}$$

where

$$\frac{de_i}{dt} \doteq \frac{\partial e_i}{\partial t} + i[H, e_i]. \tag{18}$$

Efficiency requires $\kappa t >> 1$ while accuracy is achieved if $(\kappa t)^2 << \frac{\kappa}{\langle \dot{e}_i \rangle_{\rho_0}}$. To monitor effectively and accurately fast processes we must therefore take $\kappa << 10^2 \langle \dot{e}_i \rangle_{\rho_0}$. Suppose now that H and e_i does not depend on time. Then it is easy to show that if either $\rho_0(0)$ or e_i commutes with H, we get $\tilde{p}_i(t) = q_i$ exactly and instantly.

In [3, 4] we describe and analyze a Stern Gerlach experiment and a model of a counter for a one-dimensional ultra-relativistic quantum mechanical particle.

3 Quantum Zeno Effect

Zeno of Elea is famous for the paradoxes whereby, in order to recommand the doctrine of the existence of "the one" (i.e. indivisible reality) he sought to controvert the common-sense belief in the existence of "the many" (i.e. distingnishable quantities and things capable of motion). The quantum Zeno effect was described many years ago when it was claimed that is possible to inhibit or even to stop the decay of an unstable quantum mechanical system by performing a sequence of frequent measurements. The exponential decay law $P(t) = e^{-\gamma t}$ is experimentally confirmed for most unstable particles and nuclei in a wide range of time. The initial decay rate is in this case $-\frac{dP}{dt}(0_+) = \gamma$. On the other hand, from quantum theory, we get for the decay law $\dot{P}_\psi(0) = 2Im < \psi, H\psi >= 0$ with $P_\psi(t) = | < \psi, e^{-itH}\psi > |^2$. When the particle is observed at $t/n, 2t/n, \ldots$ then $P_\psi(t) = P_\psi(t/n)^n$; now if $\dot{P}_\psi(0_+) = 0$ if follows that $\lim_{n\to+\infty} P_\psi(t/n)^n = 1$, which implies that frequent observations freeze the system in its initial state.

Using our model of a continuous measurement we can easily discuss this effect for a quantum spin 1/2 system coupled to a 2-state classical system [11]. We consider only one orthogonal projector e on the Hilbert space $\mathcal{H}_q = \mathbf{C}^2$. To specify the coupling dynamics we choose the coupling operator V in the following symmetric way:

$$X = \sqrt{\kappa}\begin{pmatrix} 0, & e \\ e, & 0 \end{pmatrix}. \tag{19}$$

The Liouville equation for the total state $\rho = diag(\rho_0, \rho_1)$ reads now

$$\dot{\rho}_0 = -i[H, \rho_0] + \kappa(e\rho_1 e - \tfrac{1}{2}\{e, \rho_0\}), \tag{20}$$

$$\dot{\rho}_1 = -i[H, \rho_1] + \kappa(e\rho_0 e - \tfrac{1}{2}\{e, \rho_1\}). \tag{21}$$

The partial quantum state $\pi_q(\rho) = \hat{\rho} = \rho_0(t) + \rho_1(t)$ evolves in this particular model independently of the state of the classical system, which expresses the fact that we have here only transport of information from $\sum_q$ to $\sum_c$. The time evolution of $\hat{\rho}(t)$ is given by

$$\dot{\hat{\rho}} = -i[H, \hat{\rho}] + \kappa(e\hat{\rho}e - \frac{1}{2}\{e, \hat{\rho}\}). \tag{22}$$

Let us now choose the Hamiltonian part $H = \frac{\omega}{2}\sigma_3$, and $e = \frac{1}{2}(\sigma_0 + \sigma_1)$, and start with the quantum system $\sum_q$ being for $t = 0$ in the eigenstate of σ_1. We repeatedly check with "frequency" κ if the system is still in this initial state, each "yes" inducing a flip in the coupled classical device, which we continuously observe. The solution of (23) such that $\hat{\rho}(0) = e$ can be easily found. Moreover it is possible for strongly coupled system i.e. for $\kappa t >> 1$ and $\kappa/\omega >> 1$ to obtain asymptotic formulae for the distance travelled by the quantum state $d(\hat{\rho}(t), e)$ in the Bures or in the Frobenius norm $\| \hat{\rho} \|^2 = Tr(\hat{\rho}^2)$. In this asymptotic regime we can show that the Bures distance achieved during the coupling is given by

$$d(\hat{\rho}(t), e) \approx \omega\sqrt{t/\kappa} \tag{23}$$

The effect of slowing down the evolution of the quantum system can be confirmed by an independent, strong but non-demolishing, coupling of a third classical device. In [4, 13] we show moreover that a piecewise deterministic Markov process taking values on pure states of the total system is naturally associated to the Liouville equation and that the coupling constant κ is the average frequency of jumps of the classical system between its two states.

Remarks on "meaning of the wave function" It is tempting to use the Zeno effect for slowing down the time evolution in such a way, that the state of a quantum system Σ_q can be determined by carrying out measurements of sufficiently many observables. This idea, however, would not work, similarly like would not work the proposal of "protective measurements" of Y. Aharonov et al (see [16] [17]). To apply Zeno-type measurements just as to apply a "protective measurement" one would have to know the state beforhand. Our results suggest that obtaining a reliable knowledge of the quantum state may necessarily lead to a significant, irreversible disturbance of the state. This negative statement does not mean that we have shown that the quantum state cannot be objectively determined. We believe however that dynamical, statistical and information-theoretical aspects of the important problem of obtaining a "*maximal reliable knowledge ;, of the unknown quantum state with a least possible disturbance*" are not yet sufficiently understood.

4 SQUID - Tank circuit interaction

Superconductivity was discovered 1911 by Kamerlingh Onnes. Two important properties of superconductors set them apart from normal metallic conductors; they exhibit zero electrical resistance to current flow and they expel magnetic fields (the Meissner effect). In addition superconductors display a special characteristic when two are coupled through a thin insulating layer (the Josephson effect). Josephson devices consist of two superconducting films through which electrons can tunnel from one superconductor to the other. The tunneling can be by superconducting pairs via the Josephson effect. In a Josephson junction the current I is given by $I = I_c \sin\phi$, where I_c is the dissipationless current that the junction will sustain. The Josephson energy E, which is the kinetic energy of the current I flowing through the junction is given by $E = -\frac{\hbar}{2e} I_c \cos\phi$ and ϕ is the quantum mechanical phase difference across the function.

A SQUID is a ring-shaped superconducting circuit containing one or more so called weak links whose behaviour is governed by the Josephson equations of superconductivity. A magnetic field applied to a SQUID alters its electrical characteristics. The ring's response can be interrogated with conventional electronics. SQUIDS posses a wide variety of macroscopic quantum mechanical properties. In recent years these has been considerable discussion of the dynamics of a system consisting of a SQUID coupled to a dissipative classical linear oscillator [18, 19, 20, 21]. Our aim is to show that a continuous version of our framework is very well adapted to discuss the behaviour of the coupled system consisting of a macroscopic classical system (tank circuit) and a single macroscopic quantum object (SQUID).

4.1 SQUID coupled to a damped classical oscillator

A SQUID (Superconducting Quantum Interference Device) consists of a piece of superconductor with two holes that nearly connect at the "weak link". Suppose now that the device is in a state where a current is flowing round one of the holes and induces therefore a magnetic field whose field lines pass through this hole. The magnetic flux in such a ring is quantized, where the "flux quantum" is given by $\frac{h}{2e}$. Under the assumption that the circulating current is very small there is only one flux quantum say in the left hole. A macroscopically distinct state would be the symmetric case where one flux quantum is localized in the right hole. Quantum theory says that a SQUID can exits also in a state where the flux is delocalized between the two holes; flux quanta can pass from one hole to another by quantum tunnelling processes. SQUIDS are laboratory versions of Schrödinger's cat. The flux Φ trapped through the ring is a macroscopic variable which obeys a standard Schrödinger equation with the mass M replaced by the capacitance of the Josephson junction and the potential $V(\Phi)$ such that $\lim_{|\Phi|\to\infty} V(\Phi) = +\infty$. Our aim is to describe the interaction between a SQUID and a classical damped oscillator. Both are macroscopic electromagnetic circuits. The classical system can be seen as a model for a local environment for the SQUID. The Hamiltonian of the quantum system contains a source term through wich it can be coupled to the classical device.

The radiofrequency (rf) SQUID is a superconducting loop which is interrupted by a thin insulating layer (Josephson tunnel junction). The conduction electrons in the superconductor are paired, and thus encounter negligible dissipation within the body of the superconductor. The Cooper pairs tunnel through the insulating barrier. In general there is some dissipation associated with this process, as well as a capacitance determined by the geometry of the junction. A superconducting screeming current I_S flows around the SQUID loop inductance L in response to an externally applied magnetic flux ϕ_{ext} generated by a magnetic field orthogonal to the SQUID for suitably chosen device parameter, the net flux obeys an equation of motion similar to that of a particle moving in a double well potential, with the capacitance C and conductance $1/R$ corresponding to the particle mass and dissipation

$$C\ddot{\Phi} + \frac{1}{R}\dot{\Phi} = -\frac{\partial V}{\partial \Phi}. \tag{24}$$

The potential V is given as a function of the net flux and the external flux Φ_{ext}

$$V(\Phi) = \frac{1}{2\Lambda}(\Phi - \phi_{ext})^2 - \frac{I_0\Phi_0}{2\pi}\cos 2\pi\frac{\Phi}{\Phi_0}. \tag{25}$$

At a flux of $\Phi_0/2$ the screening is equal in magnitude in both wells, but opposite in direction. This is the optimal point for the observation of coherent tunneling effects. The net flux Φ may be considered to be a macroscopic variable since the condensate of superconducting pairs is described by the product of a large number of pair wave functions. The large overlap of the wave functions of the Cooper pairs, which are single quantum states extending over a macroscopic distance, produces a macroscopic phase coherence. SQUIDS are devices exhibiting quantized flux but being describable by classical quantities such as voltage and current. The total magnetic flux Φ is conjugate to the total electric flux i.e. the charge Q across the weak link. Then satisfy the commutation relation

$$[\Phi, Q] = i\hbar\mathbf{1}. \tag{26}$$

The Hamilton operator for the SQUID-tank model is given by

$$H(\varphi) = \frac{Q^2}{2C} + \frac{(\Phi - \phi_{ext})^2}{2\Lambda} - \hbar\omega\cos\left(\frac{2\pi\Phi}{\Phi_0}\right) \tag{27}$$

where

$$\phi_{ext} = \varphi_{ext} + \mu\varphi \tag{28}$$

$$Q = -i\hbar\frac{d}{d\Phi} \tag{29}$$

$$\Phi_0 = \frac{h}{2e}. \tag{30}$$

For the tank equation following Spiller et al. [19] one obtains

$$\ddot{\varphi} + \frac{\dot{\varphi}}{R_t C_t} + \frac{\varphi}{L_t C_t} = \frac{1}{C_t}\left(I_{\mathbf{IN}}(t) + \mu < \frac{\Phi - \phi_{ext}}{\Lambda} > \right). \tag{31}$$

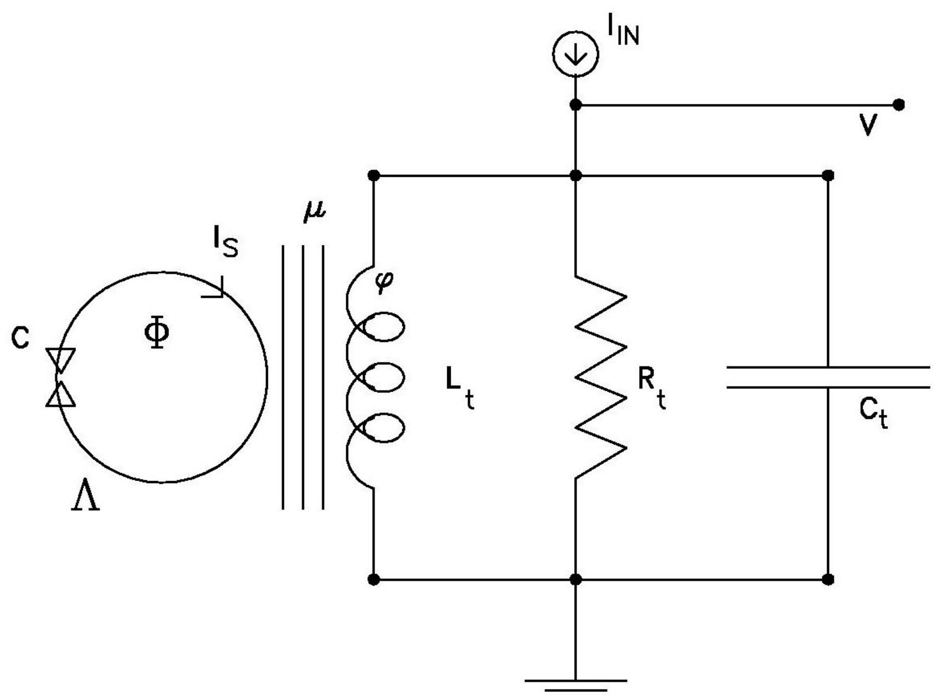

Figure 1:

States of the classical system are probabilistic measures p on its phase space $\Omega = (\mathbf{R}^2, d\varphi d\pi)$; we take for the canonical variables: the magnetic flux φ and its rate of change π (thought of as $\dot{\varphi}$). Because of dumping in the tank circuit, the equation of motion for φ cannot be written in a standard Hamiltonian form (but it can be written using a complex Hamiltonian – see e.g. [22]). For our purpose we need only the Liouville equation – which is just continuity equation for the classical flow:

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial \varphi}(\dot{\varphi} p) + \frac{\partial}{\partial \pi}(p\dot{\pi}) = 0. \tag{32}$$

Denoting $I_S := < \frac{\phi - \phi_{ext}}{\Lambda} >$, with $\quad \dot{\varphi} \to \pi, \quad \dot{\pi} \to \ddot{\varphi} \quad$ we obtain

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial \varphi}(\pi p) + \frac{\partial}{\partial \pi}\left(-p\left(\frac{\pi}{R_t C_t} + \frac{\varphi}{L_t C_t} + \frac{1}{C_t}\left(I_{\mathbf{N}} + \mu < I_S >\right)\right)\right) \tag{33}$$

or $\dot{p} = L_{cl} p$ where

$$\begin{aligned} L_{cl} p &= -\pi \frac{\partial p}{\partial \varphi} + \left\{ \frac{1}{R_t C_t}\pi + \frac{1}{L_t C_t}\varphi - \frac{1}{C_t}(I_{\mathbf{IN}} + \mu < I_S >) \right\} \times \\ &\quad \times \frac{\partial p}{\partial \pi} + \frac{1}{R_t C_t} p \ . \end{aligned} \tag{34}$$

States of the total system consisting of SQUID and tank are described by measures $\rho(\varphi, \pi)$ on Ω with values in positive trace class operators in $\mathcal{H}_q = L^2(\mathbf{R}, d\Phi)$, normalized by

$$\int Tr(\rho(\varphi, \pi)) d\varphi \, d\pi = 1 \ . \tag{35}$$

It is convenient in the spirit of Section 2 to introduce the Hilbert space $\mathcal{H}_{tot}$ for the total system:

$$\mathcal{H}_{tot} = \int^{\oplus} \mathcal{H}_q d\varphi \, d\pi. \tag{36}$$

The vector state of $\mathcal{H}_{tot}$ are then given by functions $\Psi : (\varphi, \pi) \longmapsto \Psi(\varphi, \pi) \in \mathcal{H}_q$.

The coupling between SQUID and tank is postulated to be given by the following operator (generalizing obviously the Lindblad form to a continuous family)

$$L_{int}\, \rho = \int_{-\infty}^{+\infty} da (V_a \rho V_a^* - \frac{1}{2}\{V_a^* V_a, \rho\}) \tag{37}$$

with V_a given by

$$(V_a \Psi)(\varphi, \pi) = f(\Phi - \phi_{ext} - a)\Psi(\varphi, \pi - ka) \ . \tag{38}$$

The function f can be thought of as defining a sensitivity window - it should be odd or even:

$$f(x) = \pm f(-x) \ . \tag{39}$$

We denote

$$\alpha = \int_{-\infty}^{+\infty} f^2(x) dx \ . \tag{40}$$

The constant k (having dimension $[time]^{-1}$) is the second constant characterizing the coupling. We first notice that

$$\int_{-\infty}^{+\infty} V_a^* V_a da = \alpha \cdot I \tag{41}$$

so that

$$\begin{aligned} (L_{int}\rho)(\varphi,\pi) &= \int da f(\Phi - \phi_{ext} - a)\rho(\varphi, \pi - ka) f(\Phi - \phi_{ext} - a) \\ &\quad -\alpha\rho(\varphi,\pi) \ . \end{aligned} \tag{42}$$

The Liouville operator for the total system is then given by a sum of three terms

$$\begin{aligned} (\mathcal{L}\rho)(\varphi,\pi) &= -i[H(\varphi), \rho(\varphi,\pi)] + \\ &\quad +(L_{cl}\rho)(\varphi,\pi) + \\ &\quad +(L_{int}\rho)(\varphi,\pi) \ . \end{aligned} \tag{43}$$

In the following we will denote by $< F >$ the average for a quantity F:

$$< F >= \int Tr(F(\varphi,\pi)\rho(\varphi,\pi)) d\varphi \, d\pi \ . \tag{44}$$

Therefore the time derivative of averages is given explicitly by

$$\begin{aligned} < \dot{F} > &= \int Tr(F(\varphi,\pi)\dot{\rho}(\varphi,\pi)) d\varphi \, d\pi = \\ &= \int Tr(F(\varphi,\pi)(\mathcal{L}\rho)(\varphi,\pi)) d\varphi \, d\pi \ . \end{aligned} \tag{45}$$

Let us compute

$$< \dot{\varphi} >= \int \varphi Tr((\mathcal{L}\rho)(\varphi,\pi)) d\varphi \, d\pi \ . \tag{46}$$

Only the classical part contributes and we get

$$< \dot{\varphi} >=< \pi > \ . \tag{47}$$

We need also to compute $< \ddot{\varphi} >=< \dot{\pi} >$

$$< \dot{\pi} >= \int \pi Tr(\mathcal{L}\rho(\varphi,\pi))d\varphi\, d\pi \ . \tag{48}$$

The quantum Hamiltonian does not contribute while the classical part gives nothing else as the RHS of the classical equations of motion:

$$-\frac{< \dot{\varphi} >}{R_t C_t} - \frac{< \varphi >}{L_t C_t} + \frac{1}{C_t} I_{IN}(t) \ . \tag{49}$$

We compute next the term coming from L_{int}:

$$\begin{aligned} &\int \pi Tr(f^2(\Phi - \phi_{ext} - a)\rho(\varphi, \pi - ka)da\, d\varphi\, d\pi \\ &\qquad -\alpha \int \pi Tr(\rho(\varphi,\pi))d\varphi\, d\pi \quad . \end{aligned} \tag{50}$$

Let us consider the first term. Changing variables $\pi - ka = \pi'$ we get

$$\int (\pi' + ka) Tr(f^2(\Phi - \phi_{ext} - a)\rho(\varphi,\pi'))da\, d\varphi\, d\pi' \ . \tag{51}$$

The π' term cancels with the last term in (50). We change also the a variables introducing a' by

$$a - \Phi - \phi_{ext} = a' \tag{52}$$

and obtain just

$$k \int Tr((a' + \Phi - \phi_{ext}) f^2(a')\rho(\varphi,\pi))da'\, d\varphi\, d\pi \ . \tag{53}$$

The term $\int a' f^2(a')$ gives zero because f^2 is assumed to be even. What remains is

$$\begin{aligned} \alpha k \int \quad & Tr((\Phi - \phi_{ext})\rho(\varphi,\pi))d\varphi\, d\pi = \\ &= \alpha k < \Phi - \phi_{ext} > \ . \end{aligned} \tag{54}$$

It follows that our evolution law for averages is compatible with that of Spiller [18] if we put

$$\alpha k = \frac{\mu}{C_t \Lambda} \ , \tag{55}$$

which fixes one of the parameters α, k in terms of the other.

For an arbitrary function $F(\Phi)$, we find that

$$\frac{d}{dt}F(\Phi) = i[H(\varphi), F(\Phi)] \tag{56}$$

so that the dissipative coupling does not influence time evolution of the SQUID flux variables. It will, however, in general, influence functions of its conjugate variable Q. In fact, we have

$$< \dot{Q} >=< i[H(\varphi), Q] > + < \delta\dot{Q} > \tag{57}$$

where

$$\begin{aligned} < \delta\dot{Q} > &= \int Tr(Qf(\Phi - \phi_{ext} - a)\rho(\varphi, \pi)f(\Phi - \phi_{ext} - a)d\varphi\, d\pi\, da \\ &\qquad -\alpha < Q >= 0 \end{aligned} \tag{58}$$

because $Qf - fQ \equiv f'$ and $f'f$ is odd, but $< \dot{Q}^2 >$ can be already $\neq 0$.

4.2 The partially deterministic stochastic process associated to SQUID-model

The total Liouville operator of the squid tank model splits as seen in Section 4.1 into 3 parts

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_{cl} + \mathcal{L}_{int} \ . \tag{59}$$

The parts $\mathcal{L}_q$ and $\mathcal{L}_{cl}$ give us deterministic motion of pure states of the quantum and of the classical system. The time evolution is subject to the following coupled system:

$$\begin{aligned} i\frac{d\Psi}{dt} &= H(\varphi)\Psi \\ \frac{d\Psi}{dt} &= \pi \\ \frac{d\pi}{dt} &= -\frac{\pi}{R_t C_t} - \frac{\varphi}{L_t C_t} \ . \end{aligned} \tag{60}$$

They give us a vector field X acting on the product of pure states of the quantum and of the classical system. We write now $\mathcal{L}_{int}$ as acting on observables

$$\int Tr(\rho(\varphi,\pi)(\mathcal{L}_{int}A)(\varphi,\pi)) = \int Tr((\mathcal{L}_{int}\rho)(\varphi,\pi)A(\varphi,\pi)) \ . \tag{61}$$

After a change of variables and using the cyclicity of the trace this term is then given by

$$\int Tr(\rho(\varphi,\pi)\left[\int f(\Phi-\phi_{ext}-a)A(\varphi,\pi+ka)f(\Phi-\phi_{ext}-a) - \alpha A(\varphi,\pi)\right] \ . \tag{62}$$

Thus

$$\begin{aligned} (\mathcal{L}_{int}A)(\varphi,\pi) &= \int da f(\Phi-\phi_{ext}-a)A(\varphi,\pi+ka)f(\Phi-\phi_{ext}-a) \\ &\quad -\alpha A(\varphi,\pi) \end{aligned} \tag{63}$$

In order to construct a PD-process we compute now time evolution of functions

$$F_A(\Psi;\varphi,\pi) = (\Psi, A(\varphi,\pi)\Psi) \tag{64}$$

we get

$$\dot{F}_A(\Psi,\varphi,\pi) = (\Psi, (\mathcal{L}_{int}A)(\varphi,\pi)\Psi) = \tag{65}$$

$$= \int da (f\Psi, A(\varphi, \pi + ka) f\Psi) - \alpha F_A(\Psi;\varphi,\pi) =$$

$$= \int da \|f\Psi\|^2 F_A\left(\frac{f\Psi}{\|f\Psi\|}, \varphi, \pi + ka\right) - \alpha\, F_A(\Psi,\varphi,\pi)$$

$$= \int da \|f\Psi\|^2 \delta(\Psi' - \frac{f\Psi}{\|f\Psi\|})\delta(\varphi'-\varphi)\delta(\pi'-\pi-ka) F_A(\Psi';\varphi',\pi') - \alpha\, F_A(\Psi,\varphi,\pi)\ .$$

We write the integral kernel as

$$\tilde{Q}(\Psi,\varphi,\pi|\Psi',\varphi',\pi') = \int da \|f\Psi\|^2 \delta\left(\Psi' - \tfrac{f\Psi}{\|f\Psi\|}\right) \times \delta(\varphi'-\varphi)\delta(\pi'-\pi-ka)\ . \tag{66}$$

We can now perform the a integration and obtain

$$\tilde{Q}(\Psi,\varphi,\pi|\Psi',\varphi',\pi') = \frac{1}{k}\|\tilde{f}\Psi\|^2 \delta\left(\Psi' - \frac{\tilde{f}\Psi}{\|\tilde{f}\Psi|}\right)\delta(\varphi'-\varphi) \tag{67}$$

where $\tilde{f} = f(\Phi - \phi_{ext} - \frac{\pi'-\pi}{k})$.

We compute next the rate function

$$\lambda_{\varphi,\pi}(\Psi) = \int d\Psi' \int d\varphi' \int d\pi'\, \tilde{Q}(\Psi,\varphi,\pi|\Psi',\varphi',\pi') = $$

$$= \frac{1}{k}\int d\pi'\, \|\tilde{f}\Psi\|^2 = \alpha\ . \tag{68}$$

Then introducing $\tilde{Q}$ by

$$Q(\Psi;\varphi,\pi|\Psi';\varphi',\pi') = \frac{\tilde{Q}}{\alpha} \tag{69}$$

we obtain for $\dot{F}_A$

$$\dot{F}_A(\Psi,\varphi,\pi) = \alpha \int Q(\Psi,\varphi,\pi|\Psi',\varphi',\pi') F_A(\Psi',\varphi',\pi') - \alpha\, F_A(\Psi,\varphi,\pi)\ . \tag{70}$$

This time evolution equation is obviously of the type discussed by Davis [23]. As a last remark we note that the partially deterministic time evolution can be described in the following way:

The system starts in the pure state $(\Psi_0, \varphi_0, \pi_0)$ and evolves deterministically – the classical system according to

$$\ddot{\varphi} + \frac{\dot{\varphi}}{R_t C_t} + \frac{\varphi}{L_t C_t} = 0 \tag{71}$$

and the quantum system according to

$$i\dot{\Psi} = H(\varphi)\Psi \tag{72}$$

until random time t_1 governed by a Poisson process with constant rate α. At time t_1 quantum state jumps to

$$\frac{f(\Phi - \phi_{ext}(\varphi) - \frac{\pi' - \pi}{k})\Psi}{\| f(\Phi - \phi_{ext}(\varphi) - \frac{\pi' - \pi}{k})\Psi \|} \tag{73}$$

and the classical system changes its state in the following way

$$\varphi \longrightarrow \varphi' \; = \; \varphi \tag{74}$$

$$\pi \longrightarrow \pi' \tag{75}$$

with probability density

$$\frac{1}{k} \| f(\Phi - \phi_{ext} - \frac{\pi' - \pi}{k})\Psi \|^2 \; . \tag{76}$$

Notice that $\varphi' = \varphi$ from which it follows that the trajectories of the classical system are continuous. Only velocity $\pi = \dot{\varphi}$ jumps at random jump times.

5 Concluding remarks

The mathematical developments constituting Quantum Mechanics have been outstandingly successful in describing and computing (although we would not say explaining) not only those phenomena for which it was invented but also numerous others making many wonderful advances in technology possible. On the other way it is fair to say that the conceptual basis of Quantum Mechanics is still somewhat obscure. The class of models we introduced seems to provide a reliable means of extracting from mathematically consistent models of information transfer from $\sum_q$ to $\sum_c$ well defined predictions for the outcome of any experiment we can envisage - apart of course from the difficulty of solving the mathematical equations, which can be intricate and sophisticated. Physics is the study of reproducible phenomena and a statistical theory of the quantum world is all that theoretical physics whould seek. But as a statistical theory Quantum Mechanics is still a deterministic theory. On the other hand recent advances in study of chaos and algorithmic randomness suggest that near future can bring essentially new elements to our understanding of randomness - both in the realm of foundations of science and in the Nature itself. Any progress in this area may influence our current quantum paradigm.

References

[1] J. S. Bell, Against "Measurement " Physics World, August 1990, 33-40.

[2] Ph. Blanchard, A. Jadczyk, On the interaction between classical and quantum systems, Physics Letters A 175 (1993) 157-164.

[3] Ph. Blanchard, A. Jadczyk, Classical and Quantum Intertwine, in Proceedings of the Symposium of Foundations of Modern Physics, Cologne, June 1993, Ed. P. Mittelstaedt, World Scientific (1993).

[4] Ph. Blanchard, A. Jadczyk, From Quantum Probabilities to Classical Facts, in Advances in Dynamical Systems and Quantum Physics, Capri, May 1993, Ed. R. Figari, World Scientific (1994).

[5] R. Haag, Irreversibility introduced on a fundamental level, Comm. Math. Phys. 123 (1990) 245-251.

[6] J. L. Sauvageot, J. P. Thouvenot, Une nouvelle définition de l'entropie dynamique des systémes non commutatifs, Comm. Math. Phys. 145 411-423 (1992).

[7] P. Busch, P. J. Lahti, P. Mittelstaedt, The Quantum Theory of Measurement, Springer (1991).

[8] M. Namiki, S. Pascazio, Quantum Theory of Measurement Based on the many-Hilbert-Space approach, Physics Reports 232 No. 6 (1993) 301-411.

[9] N. G. van Kampen, Macroscopic Systems in Quantum Mechanics, Physica A 194 (1993) 542-550.

[10] J. S. Bell, Are these quantum jumps? in Schrödinger, Centenary of a Polymath", Cambridge University Press (1987).

[11] Ph. Blanchard, A. Jadczyk, Strongly coupled quantum and classical systems and Zeno's effect, Physics Letters A 183 (1993) 272-276.

[12] A. Connes, H. Narnhofer, W. Thirring, Dynamical entropy of C^*-algebras and von Neumann algebras, Comm. Math. Phys. 112, 691-719 (1987).

[13] Ph. Blanchard, A. Jadczyk, Coupled quantum and classical systems, measurement process, Zeno effect and all that, in preparation.

[14] Ph. Blanchard, A. Jadczyk, Nonlinear effects in coupling between classical and quantum systems: SQUID coupled to a classical damped oscillator, to appear.

[15] A. Jadczyk, Topics in Quantum Dynamics, to appear in Proc. First Caribbean Spring School of Math. and Theor. Phys., Saint Francois, Guadeloupe June 1993, World Scientific, to appear

[16] Y. Aharonov, J. Anadan, L. Vaidman, Meaning of the wave function, Phys. Rev. A47 (1993) 4616-4626.

[17] Y. Aharonov, L. Vaidman, Measurement of the Schrödinger wave of a simple particle, Phys. Letters A178 (1993) 38-42.

[18] T. P. Spiller, T. D. Clark, R. J. Prance, H. Prance, The adiabatic monitoring of quantum objects, Phys. Letters A170 (1992) 273-279.

[19] T. P. Spiller, T. D. Clark, R. J. Prance, A. Widom, Quantum Phenomena in Circuits at low temperature, Progress in Low Temperature Physics, Vol. XIII (1992) 221-265.

[20] J. F. Ralph, T. P. Spiller, T. D. Clark, R. J. Prance, H. Prance, Chaos in a coupled quantum-classical system, Phys. Letters A180 (1993) 56-60.

[21] J. F. Ralph, T. P. Spiller, T. D. Clark, H. Prance, R. J. Prance, A. J. Clippingdale, Non-linear behaviour of teh rf-SQUID magnometer, Physica D63 (1993) 191-201.

[22] Dekker,H.:On the phase space quantization of the linearly damped harmonic oscillator, Physica 95 A (1979) 311 – 323

[23] Davis,M.H.A., Markov models and optimization, Chapman and Hall, London 1991