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QUANTUM MECHANICS WITH EVENT DYNAMICS†

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Event generating algorithm corresponding to a linear master equation of Lindblad's type is described and illustrated on two examples: that of a particle detector and of a fuzzy clock. Relation to other approaches to the foundations of quantum theory and to the description of quantum measurements is briefly discussed.

1. Introduction

In a recent series of papers (cf. [1] and references therein) we enhanced the standard framework of quantum mechanics endowing it with event dynamics. In this extension, which will be denoted EEQT (for Event Enhanced Quantum Theory), we go beyond the Schrödinger continuous-time evolution of wave packets — we also propose a class of algorithms generating discrete events. From master equation that describes continuous evolution of ensembles of coupled quantum + classical systems we derive a unique piecewise deterministic random process that provides a stochastic algorithm for generating sample histories of individual systems. In the present contribution we will describe the essence of our approach. But first we make a few comments on similarities and differences between EEQT and several other approaches.

(1) *The standard approach*

In the standard approach classical concepts are static. They are introduced via measurement postulates developed by the founders of Quantum Theory. But "measurement" itself is never precisely defined in the standard approach and therefore measurement postulates cannot be derived from the formalism. One is supposed to believe Born's statisti-

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cal interpretation simply "because it works". The standard interpretation alone does not tell us what happens when a quantum system is under a continuous observation (which, in fact, is always the case).

(2) *Master equation dynamics and continuous observation theory*

Continuous observation theory is usually based on successive applications of the projection postulate. Each application of the projection postulate maps pure states into mixed states. Thus repeated application of the postulate leads to a master equation for a density matrix. Replacing Schrödinger's dynamics by a master equation is also popular in quantum optics (cf. [2]) and in several attempts to reconcile quantum theory with gravity (for a recent account see [3]). In all these approaches the authors usually believe that no classical system is introduced. All is purely quantum. That is, however, not true. What is true is just the converse: the largest possible classical system is introduced, but because it is so large and so close to the eye, it easily escapes our sight. It is assumed, without any justification, that jumps of quantum state vectors are directly observable (whatever it means). These jumps are supposed to constitute the only classical events. The weak point of this approach is in the fact that going from the master equation, that describes statistical ensembles, to a stochastic algorithm generating sample histories of an individual system is non-unique. There are infinitely many random processes that lead to the same master equation after averaging. One can use diffusion stochastic differential equations or jump processes, one can shift pieces of dynamics between Hamiltonian evolution and collapse events.

The reason for this non-uniqueness is simple: there are infinitely many mixtures that lead to the same density matrix. Diosi [4, 5] invented a clever mathematical procedure for constructing a special "ortho-process". It provides a definite algorithm in special cases of finite degeneracy. It does not however remove non-uniqueness and also there is no reason why Nature should have chosen this special prescription causing quantum state vector always to make the least probable transition: to one of the orthogonal states.

(3) *Bohmian mechanics, local beables, stochastic mechanics*

In these approaches (cf. refs. [6–8]) *there is* an explicit classical system. Quantum state vector knows nothing about this classical system. It evolves according to the unmodified Schrödinger's dynamics. It acts on the classical system affecting the classical dynamics (which is either causal or stochastic) without itself being acted upon. There is a mysterious *quantum potential*: action without reaction. All such schemes are inconsistent with quantum mechanics. They can be shown to contradict indistinguishability of quantum mixtures that are described by the same density matrix [9]. That it must be so follows from quite general no-go theorems — cf. [10–12]. The fact that the above schemes allow us to distinguish between mixtures that standard quantum mechanics considers indistinguishable need not be a weakness. In fact, it may be an advantage because it may lead beyond quantum theory, it can provide us with means of faster-than-light communication — provided experiment confirms this feature.

How does our approach compare to those above? First of all, as for today, our approach is explicitly phenomenological. That is not to say that, for instance, the standard

approach is not phenomenological. In the standard approach *we* must decide where do we finish our quantum description and what do *we* "measure". That does not follow from the theory — it must be imputed from the outside. However, we have been so much indoctrinated by Bohr's philosophy and its apparent victory over Einstein's "realistic" dreams, and we are today so used to this procedure, that we do not feel uneasiness here any more. Somehow we tend to believe that the future "quantum theory of everything" will explain all events that happen. But chances are that this theory of everything will explain nothing. It will be a dead theory. It will not even have a Hamiltonian, because there will be no time. It will be a theory of the world in which nothing happens by itself. It will answer *our* questions about certain *probabilities* - when these questions are asked. But it will not explain why anything *happens* at all.

Our theory of event dynamics starts with an explicit phenomenological split between a quantum system, which is not directly observable, and a classical system, where events happen that can be observed and that are to be described and explained. In other words our starting point is an explicit mathematical formulation of Heisenberg's cut. The quantum system may be as big as one wishes it to be, the classical system may retreat more and more, moved as far as we wish — towards our sense organs, towards our brains, towards our mental processes. But the further we retreat the less *facts* we explain. At the extreme limit we will be able to explain nothing but changes of our mental states, i.e. only mental events. That state of affairs may be considered satisfactory for those who adhere to idealistic or eastern philosophies, but it need not be the one that enriches our understanding of the true workings of Nature. Probably, for most of practical purposes, it is sufficient to retreat with the quanto-classical cut as far as photon detection processes which can be treated as the primitive events. However, our event mechanics works quite well when the cut between the quantum and the classical is expressed in engineering language: like in the example of quantum SQUID coupled to a classical radio-frequency circuit, or quantum particle coupled to its yes-no position detectors, for instance to a cloud chamber.

Once the split between the quantum and the classical is fixed, then the coupling between both systems is described in terms of a special master equation. Because of its special form there is a unique random process in the space of pure states of the total system that reproduces this master equation. The process gives an algorithm for generating sample histories. It is of piecewise deterministic character. It consists of periods of continuous evolution interrupted by jumps and events that happen at random times. The continuous evolution of the quantum system is described by a — modified by the coupling — nonunitary Schrödinger's equation. The jump times have a Poissonian character, with their jump rates dependent on the actual state of both the quantum and the classical system. The back action of the classical system on the quantum one shows up in two ways: first of all by modifying the Schrödinger evolution between jumps by a non-unitary damping, second by causing quantum state to jump at event times. Notice that the master equation describing statistical properties is linear, while the evolution of individual system is nonlinear. This agrees with Turing's aphorism stating that "prediction must be linear, description must be nonlinear" [13].

Our theory, even if it works well and has a practical value, should be considered not as a final scheme of things, but merely as a step that may help us to find a description of Nature that is more satisfactory than the one proposed by the orthodox quantum philosophy. Pure quantum theory proposes a universe that is dead — nothing happens, nothing is real — apart from the questions asked by mysterious "observers". But these observers are metaphysics, are not in the equations. In a sharp contrast to the standard approach, our theory of event mechanics described here makes the universe "running" again, even before there were any observers. It has gotten however the arrow of time that is driven by a fuzzy quantum clock. It also needs a roulette. This is hard to accept for many of us. We would like to believe that Nature is ruled by a perfect order. And to be perfect - this order must be deterministic. Even if we do not share Einstein's dissatisfaction with quantum theory, we tend to understand his disgust at the very thought of God playing dice. But Nature's concept of a perfect order may be not that simple one as we wish. Perhaps using probability theory may be the only way of describing in finite terms the universe that has an infinite complexity. It may be that we will never know the ultimate secret, nevertheless the mechanism proposed by EEQT brings a hope of restoring some order that we are seeking. Namely, the quanto-classical clock that we describe below works "by itself". It is true that it needs a roulette but the roulette is a *classical* roulette. We need only *classical probability*, and classical random processes. That is good because we understand classical probability by hearts but "quantum probability" we understand only by abstract terms. That is some progress also because nowadays we know more about complexity theory, theory of random sequences, and theory of chaotic phenomena. Each year we find new ways of generating apparently random phenomena out of deterministic algorithms of sufficient complexity. In fact, our event generating algorithm is successfully simulated with a completely deterministic classical computer. The crucial problem here is the necessary computing power. Moreover, the algorithm is nonlocal. We do not know how Nature manages to make its world clock running with no or little effort. We must yet learn it.

2. The event engine

We will describe in this section the event generating algorithm that results from our theory. The algorithm is simple, the master equation that it leads to is also easy to write down. What is more difficult is proving that the correspondence between statistical description provided by the master equation implies the event algorithm uniquely, cf. [1]. To make the idea as clear as possible we will assume that our classical system admits only finite number of states. We will call these states $\alpha = 1, \ldots, m$. There are $m^2 - 1$ possible events labeled by pairs $\alpha \neq \beta$. For each α let $\mathcal{H}_\alpha$ be the Hilbert space of the quantum system. Usually all these Hilbert spaces are isomorphic or even identical. But it costs us nothing to allow for a more general setting, so that the transition $\alpha \to \beta$ may correspond to phase transition, where Hilbert space must also change. We then need m^2 operators (or $m^2 - m$ operators in a symmetric case, see below): m Hermitian operators

H_α — the Hamiltonians $H_\alpha : \mathcal{H} \to \mathcal{H}_\alpha$, and $m^2 - m$ operators $g_{\alpha\beta} : \mathcal{H}_\beta \to \mathcal{H}_\alpha$. Thus our operator-valued matrix $g_{\alpha\beta}$ has zeros on the diagonal. The theory becomes most symmetric if the so called "detailed balance condition" is satisfied, that is if $g^*_{\alpha\beta} = g_{\beta\alpha}$. But there are no reasons to impose this condition and our two examples in the next section are not symmetric. The operators $H_\alpha, g_{\alpha\beta}$ may depend explicitly on time. We will not make this dependence explicit but all our formulae below are written in such a way that they remain valid in this more general case.

Before describing our event generating process let us introduce a convenient notation: for any $\psi_\alpha \in \mathcal{H}_\alpha$ denote

$$\Lambda_\alpha = \sum_\beta g^\star_{\beta\alpha} g_{\beta\alpha}\,, \tag{1}$$

$$\lambda_\alpha(\psi_\alpha) = (\psi_\alpha, \Lambda_\alpha \psi_\alpha)\,, \tag{2}$$

$$p_\beta(\psi_\alpha) = \frac{\|g_{\beta\alpha}\psi_\alpha\|^2}{\lambda_\alpha(\psi_\alpha)}\,. \tag{3}$$

2.1. Event generation

The algorithm powering our event engine is described by the following steps:

Event Generating Algorithm:

Given on input t_0, α_0 and $\psi_0 \in \mathcal{H}_{\alpha_0}$, with $\|\psi_0\| = 1$, it produces on output t_1, α_1 and $\psi_1 \in \mathcal{H}_{\alpha_1}$, with $\|\psi_1\| = 1$.

(1) *Choose uniform random number* $r \in [0,1]$.

(2) *Propagate* ψ_0 *in* $\mathcal{H}_{\alpha_0}$ *forward in time by solving*:

$$\dot{\psi} = \left(-iH_{\alpha_0} - \frac{1}{2}\Lambda_{\alpha_0}\right)\psi \tag{4}$$

with initial condition $\psi(t_0) = \psi_0$ *until* $t = t_1$, *where* t_1 *is defined by*

$$\|\psi(t_1)\|^2 = r\,. \tag{5}$$

(3) *Choose uniform random number* $r' \in [0,1]$.

(4) *Run through the classical states* $\beta = 1, 2, \ldots, m$ *until you reach* $\beta = \alpha_1$ *for which*

$$\sum_{\beta=1}^{\alpha_1} p_\beta(\psi(t_1)) \geq r'\,. \tag{6}$$

(5) *Set* $\psi_1 = g_{\alpha_1\alpha_0}\psi(t_1)/\|g_{\alpha_1\alpha_0}\psi(t_1)\|$.

Time evolution of an individual system is described by repeated application of the above algorithm, using its output as the input for each next Step. If we want to study time evolution in a given interval $[t_{\text{in}}, t_{\text{fin}}]$, then we apply the algorithm by starting with

$t_0 = t_{\text{in}}$, repeating it until we reach $t = t_{\text{fin}}$ somewhere in the middle of propagation in Step 2. Then we normalize the resulting state.

Remark 1: According to the theory developed in [1] the jump process is an inhomogeneous Poisson process with intensity function $\lambda_\alpha(t)$, α being the actual state of the classical system. One way to simulate such a process is to move forward in time by small time intervals Δt, and make independent decisions for jumping with probability $\lambda_\alpha(t)\Delta t$. This leads to the probability p of the first jump to occur in the time interval (t_0, t) given by:

$$p = 1 - \exp\left(-\int_{t_0}^{t} \lambda_\alpha(s)\, ds\right). \tag{7}$$

By using the identity $\log f(t) - \log f(t_0) = \int_{t_0}^{t} \dot{f}(s)/f(s)\, ds$ it is easy to see that $p = 1 - \|\psi_\alpha(t)\|^2$ — which simplifies simulation — as we did in Steps 2, 3 above. This observation throws also some new light on those approaches to quantum mechanical description of particle decays that were based on non-unitary evolution.

Remark 2: The algorithm above involves playing a roulette. If Nature is using this algorithm running Her event engine, then the timing of each next event is decided beforehand in Step 2. But even if r is already chosen, still there is a possibility to delay or to hasten the next event provided one has the ability to manipulate the time-dependence of $g_{\beta\alpha}(t)$ that enter $\Lambda_\alpha(t)$ in Eq. (5). In other words, it is by using $g_{\beta\alpha}(t)$ tools we can, to some extent, manipulate Moira.

2.2. Master equation

By repeating the above event generating algorithm many times, or by observing time series of events for a prolonged time, we will notice certain regularities and certain statistical tendencies. There are many ways of collecting data that we consider of interest. For instance, we may ask what is the average time necessary for arriving at a particular classical state or a succession of states. But we can also ask more standard question: suppose we repeat our simulation many times always starting with the same state at the same initial time t_0, and ending it at the same final time t. Then we will arrive at different final states with different probabilities. Let α, ψ_α, t_0 be the initial state, and let $\mu(\alpha, \psi_\alpha, t_0; \beta, \psi_\beta, t)$ be the probability density of arriving at the state (β, ψ_β) at time t. We may associate with this probability distribution a family of density matrices:

$$\rho_\beta = \int \mu(\alpha, \psi_\alpha, t_0; \beta, \psi_\beta, t) |\psi_\beta\rangle\langle\psi_\beta|\, d\psi_\beta\,, \tag{8}$$

so that $\sum_\beta \mathrm{Tr}\rho_\beta = 1$. This association is many to one. We lose information this way. Nevertheless, as shown in [1, 14], the following theorem holds:

THEOREM 1. *The family $\rho_\beta(t)$ satisfies linear differential equation*

$$\dot{\rho}_\beta = -i[H_\beta, \rho_\beta] + \sum_{\gamma \neq \beta} g_{\beta\gamma}\rho_\gamma g^*_{\beta\gamma} - \frac{1}{2}\{\Lambda_\beta, \rho_\beta\}\,, \tag{9}$$

where $\{\ ,\ \}$ *stands for anticommutator. Conversely, the process with values in pure states* (α, ψ_α) *described in the previous subsection is the unique one leading to* (9).

Equation (9) describes time evolution of statistical states of the total, classical + quantum system. Sometimes, in special cases, it is possible to sum up over β to obtain evolution equation for the effective statistical state of the quantum system alone. For this being possible first of all the Hilbert spaces $\mathcal{H}_\beta$ must be identical. Then we can set $\rho = \sum_\beta \rho_\beta$. Also, we must have the same Hamiltonian, and the same Λ in each "channel": $H_\beta \equiv H, \Lambda_\beta \equiv \Lambda$, moreover we must have special property that $\sum_\beta \sum_\alpha g_{\beta\gamma}\rho_\gamma g^*_{\beta\gamma} = \sum_i V_\subset \rho V_i^*$ for some family of operators V_i which result from summing up subfamilies of operators $g_{\beta\gamma}$. Only in that case we obtain Liouville's evolution equation for ρ:

$$\dot{\rho} = -i[H, \rho] + \sum_i V_i \rho V_i^* - \frac{1}{2}\{\Lambda, \rho\}\ , \tag{10}$$

with $\Lambda = \sum_i V_i^* V_i$. But even if this is the case, the information lost is unrecoverable: there are always infinitely many processes in the space of pure states of the quantum system that lead to the same quantum master equation (10). Even if equations (9) and (10) look similar in form, there is an abyss of information loss that separates their contents.

3. Examples

3.1. Particle detector

We consider the simplest case: that of a two-state classical system. We call its two states "on" and "off". Its action is simple: if it is off, then it will stay off forever. If it is on, then it can detect a nearby particle and go off. Later on we will specialize to the detection of particle presence at a given location in space. For a while let us be general and assume that we have two Hilbert spaces $\mathcal{H}_{\text{off}}, \mathcal{H}_{\text{on}}$ and two Hamiltonians $H_{\text{off}}, H_{\text{on}}$. We also have time-dependent family of operators $g_t : \mathcal{H}_{\text{on}} \to \mathcal{H}_{\text{off}}$ and let us denote $\Lambda_t = g_t g_t^* : \mathcal{H}_{\text{on}} \to \mathcal{H}_{\text{on}}$. According to the theory presented in the previous section, with $g_{\text{off,on}} = g_t$, $g_{\text{on,off}} = 0$, the master equation for the total system, i.e. for particle and detector, reads

$$\begin{aligned} \dot{\rho}_{\text{off}}(t) &= -i[H_{\text{off}}, \rho_{\text{off}}(t)] + g_t \rho_{\text{on}}(t) g_t^* \\ \dot{\rho}_{\text{on}}(t) &= -i[H_{\text{on}}, \rho_{\text{on}}(t)] - \frac{1}{2}\{\Lambda_t, \rho_{\text{on}}(t)\}. \end{aligned} \tag{11}$$

Suppose at $t = 0$ the detector is "on" and the particle state is $\psi(0) \in \mathcal{H}_{\text{on}}$, with $\|\psi(0)\| = 1$. Then, according to the event generating algorithm described in the previous section, the probability of detection during time interval $(0, t)$ is equal to $1 - \|\exp(-iH_{\text{on}}t - \frac{t}{2}\Lambda_t)\,\psi(0)\|^2$.

Let us now specialize and consider a detector of particle presence at a location a in space (of n dimensions). Our detector has a certain range of detection and certain

efficiency. We encode these detector characteristics in a gaussian function

$$g(x) = \kappa^{1/2} \left(\frac{d}{\sigma\sqrt{\pi}} \right)^{n/2} \exp\left(\frac{-x^2}{2\sigma^2} \right), \tag{12}$$

where d, κ are constants, σ is a width parameter, and n stands for the number of space dimensions. If the detector is moving in space along some trajectory $a(t)$, and if the detector characteristics are constant in time and space, then we put: $g_t(x) = g(x - a(t))$. Let us suppose that the detector is off at $t = t_0$ and that the particle wave function is $\psi_0(x)$. Then, according to the algorithm described in the previous section, the probability of detection in the infinitesimal time interval $(t_0, t_0 + \Delta t)$ equals $\int g_{t_0}^2(x)|\psi_0(x)|^2 dx \cdot \Delta t$. In the limit $\sigma \to 0$, when $g_t^2(x) \to (\kappa/d^n)\delta(x - a(t))$ we get $(\kappa/d^n)|\psi_0(a(t_0))|^2 \cdot \Delta t$. Thus we recover the usual Born interpretation, with the evident and necessary correction that the probability of detection is proportional to the length of exposure time of the detector.

That simple formula holds only for short exposure times. For a prolonged detection the formula becomes more involved, mainly because of nonunitary evolution due to the presence of the detector. In that case numerical simulation is necessary. To get an idea of what happens let us consider a simplified case which can be solved exactly. We consider the ultra-relativistic Hamiltonian $H = -icd/dx$ in space of one dimension, c being a constant. In that case the non-unitary evolution equation is easily solved:

$$\psi(x,t) = \exp\left\{ -\frac{1}{2} \int_0^t \Lambda_s(x + c(s - t)) \right\} \psi(x - t, 0)\,. \tag{13}$$

In the limit $\sigma \to 0$ when detector shrinks to a point (but becomes more and more sensitive), and assuming that this point is fixed in space $a(t) = a$, we obtain for the probability $p(t)$ of detecting the particle in the time interval $(0, t)$:

$$p(t) = \text{const} \times \int_{a-ct}^{a} |\psi(x,0)|^2 dx\,. \tag{14}$$

Intuitively this result is very clear. Our Hamiltonian describes a particle moving to the right with velocity c, the shape of the wave packet is preserved. Then $p(t)$ is equal to the standard quantum mechanical probability that the particle at $t = 0$ was in a region of space that guaranteed passing the detector, multiplied by the detector efficiency factor.

3.2. Fuzzy clock

This example illustrates diversity of possible couplings between a classical and a quantum system. In the model below no information is transferred from the quantum system, except that the passing of fuzzy units of time is marked. The example also shows that the standard continuous unitary evolution of quantum mechanics can be approximated with an arbitrary accuracy by a pure jump process.

Again, as in the subsection above we will start with a setting which is more general than usual — we will work with a family of Hilbert spaces rather than with one fixed

Hilbert space. Those readers who like to have only one Hilbert space may think that all our $\mathcal{H}_i$ below are identical to some standard Hilbert space $\mathcal{H}$.

Remark: The situation here is similar to that of a relativistic Dirac's equation. There is a separate Hilbert space for each space-like hypersurface, namely the Hilbert space of Cauchy data. There are different possibilities to identify these Hilbert spaces: different coordinate systems used by different observers will lead to different identifications. Similar situation occurs in Galilean general relativistic quantum mechanics, see [15].

For the classical system we take the set of clock readings, i.e. the set $\mathbb{Z}$ of integers i. For each i we have a Hilbert space $\mathcal{H}_i$. As we have already said before, there is no Hamiltonian part in the evolution. Concerning the classical events: the only events that we admit are clock's ticks. To each event $i-1 \to i$ we associate the operator $g_{i,i-1} = \sqrt{\kappa} U_i$, where U_i is an isometry from $\mathcal{H}_{i-1}$ to $\mathcal{H}_i$. Thus $U_i^* U_i = I$ and our master equation (9) reads

$$\dot{\rho}_i = U_i \rho_{i-1} U_i^* - \kappa \rho_i \,. \tag{15}$$

The associated process is of the simplest possible kind: at random times, distributed according to the Poisson law with a constant rate κ, the quantum state vector changes

$$\mathcal{H}_{i-1} \ni \psi_{i-1} \longrightarrow \psi_i \;=\; U_i \psi_{i-1} \in \mathcal{H}_i \,, \tag{16}$$

and the classical clock pointer advances by one $i \to i+1$. The clock is fuzzy and its clicks are random. If we want to count time more uniformly we must collect large number of such clocks. But that is not the point here. The main point of this example is to illustrate our thesis: *no dissipation — no information.* Indeed, there is no dissipation in the quantum system in this example. Quantum pure states evolve into quantum pure states. At the same time we learn nothing useful by observing the classical system. We just learn that time has passed. And this passage of time brings no information whatsoever about the quantum state. The clock rate is constant: it is completely independent of the quantum state.

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