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Events and piecewise deterministic dynamics in event-enhanced quantum theory

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Abstract

We enhance the standard formalism of quantum theory to enable events. The concepts of experiment and of measurement are defined. Dynamics is given by Liouville's equation that couples a quantum system to a classical one. It implies a *unique* Markov process involving quantum jumps, classical events and describing sample histories of individual systems.

1. Introduction

We start with recalling Bell's opinion on quantum measurements. He studied the subject in depth and he concluded, emphasizing it repeatedly [1,2]: our difficulties with quantum measurement theory are not accidental – they have a reason. He has pointed out this reason: it is that the very concept of "measurement" *cannot even be precisely defined within the standard formalism.* We agree, and we propose a way out that has not been tried before. Our scheme solves the essential part of the quantum measurement puzzle – it gives a unique algorithm generating time series of pointer readings in a continuous experiment involving quantum systems. We do not pretend that our solution is the only one that solves the puzzle. But we believe that it is a kind of a minimal solution. Even if not yet complete, it may help us to find a way towards a more fundamental theory.

The solution that we propose does not involve hidden variables. First, we point out the reason *why* "measurement" cannot be defined within the standard approach. That is because the standard quantum formalism has no place for "events" . The only candidate for an event that we could think of – in the standard formalism – is a change of the quantum state vector. But one cannot see state vectors directly. Thus, in order to include events, we have to extend the standard formalism. That is what we do, and we are doing it in a minimal way: just enough to accommodate classical events. We add explicitly a classical part to the quantum part, and we couple classical to quantum. Then we define "experiments" and "measurements" within the so extended formalism. We can show then that the standard postulates concerning measurements – in fact, in an enhanced and refined form – can be derived instead of being postulated.

This "event enhanced quantum theory" or EEQT, as we call it, gives experimental predictions that are stronger than those obtained from the standard theory. The new theory gives answers to more experimental

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questions than the old one. It provides algorithms for numerical simulations of experimental time series obtained in experiments with single quantum systems. In particular this new theory is falsifiable. We are working out its new consequences for experiments, and we will report the results in due time. But even assuming that we are successful in this respect, even then our program will not be complete. Our theory, in its present form, is based on an explicit selection of an "event carrying" classical subsystem. But how do we select what is classical? Is it our job or is it Nature's job? When we want to be on a safe side as much as possible, or as long as possible, then we tend to shift the "classical" into the observer's mind. That was von Neumann's way out. But if we decide to blame mind – shall we be safe then? For how long? It seems, not too long. This is the age of information. Soon we will need to extend our physical theory to include a theory of mind and a theory of knowledge. That necessity will face us anyhow, perhaps even sooner than we are prepared to admit. But, back to our quantum measurement problem, it is not clear at all that the cut must reside that far from the ordinary, "material" physics. For many practical applications the measuring apparatus itself, or its relevant part, can be considered classical. We need to derive such a splitting into classical and quantum from some clear principles. Perhaps is is a dynamical process, perhaps the classical part is growing with time. Perhaps time is nothing but accumulation of events. We need new laws to describe dynamics of time itself. At present we do not know what these laws are, we can only guess.

At the present stage placement of the split is indeed phenomenological, and the coupling is phenomenological too. Both are simple to handle and easy to describe in our formalism. But where to put the Heisenberg cut – that is arbitrary to some extent. Perhaps we need not worry too much? Perhaps relativity of the split is a new feature that will remain with us. We do not know. That is why we call our theory "phenomenological". But we would like to stress that the standard, orthodox, pure quantum theory is not better in this respect. In fact, it is much worse. It is not even able to define what measurement is. It is not even a phenomenological theory. In fact, strictly speaking, it is not even a theory. It is partly an art, and that needs an artist. In this case it needs a physicist with his human experience and with his human intuition. Suppose we have a problem that needs quantum theory for its solution. Then our physicist, guided by his intuition, will replace the problem at hand by another problem, that can be handled. After that, guided by his experience, he will compute Green's function or whatsoever to get formulas out of this other problem. Finally, guided by his previous experience and by his intuition, he will interpret the formulas that he got, and he will predict some numbers for the experiment.

That job cannot be left to a computing machine in an unmanned space-craft. We, human beings, may feel proud that we are that necessary, that we cannot be replaced by machines. But would it not be better if we could spare our creativity for inventing new theories rather than spending it unnecessarily for application of the old ones?

In this Letter we put stress only on the essential ideas. Details will appear in Ref. [3], where an extensive list of references, as well as many credits to earlier work by other authors, are given.

1.1. Summary of the results

In this subsection we summarize the essence of our approach. Using informal language EEQT can be described as follows:

Given a "wavy" quantum system $\mathcal{Q}$ we allow it to generate distinct classical traces – *events*. Quantum wave functions are not directly observable. They may be considered as hidden variables of the theory. On the other hand, events are discrete, in principle observable directly, real. Typically one can think of detection events and pointer readings in quantum mechanics, but also of creation–annihilation events in quantum field theory. They can be observed but they do not need an observer for their generation (although *some* may be triggered by an observer's participation). They are either recorded or they are causes for other events. It is convenient to represent events as changes of state of a suitable classical system. Thus formally we divide the world into $\mathcal{Q} \times \mathcal{C}$ – the quantum and the classical part. They are coupled together via a specific dynamics that can be encoded in an irreversible Liouville evolution equation for statistical states of the total $\mathcal{Q} \times \mathcal{C}$ system. To avoid misunderstanding we wish to stress it rather strongly: the fact that $\mathcal{Q}$ and $\mathcal{C}$ are coupled by a dissipative irreversible rather than by unitary reversible dynamics *does not mean* that noise, or heat, or chaos, or environment, or lack of knowledge, are involved. In fact, each of these factors, if present – and all of them are present in real circumstances – only blurs out transmission of information between $\mathcal{Q}$ and $\mathcal{C}$. The fact that $\mathcal{Q}$ and $\mathcal{C}$ must be coupled by a dissipative rather than by reversible dynamics follows from no-go theorems that are based on rather general assumptions [4–6]. We go beyond these abstract no-go theorems that are telling us what *is not* possible. We look for what *is* possible, and we propose a class of couplings that, as we believe, is optimal for the purposes of control and measurement. With our class of couplings no more dissipation is introduced than it is necessary for transmission of information from $\mathcal{Q}$ to $\mathcal{C}$. Thus our Liouville equation that encodes the measurement process is to be considered as *exact*, not as an approximate one (adding noise to it will make it approximate). Given such a coupling we can show that the Liouville equation encodes in a unique way the algorithm for generating admissible histories of individual systems. That part is new comparing with our previous paper [7]. While writing Ref. [7] we did not know how to describe individual systems. We did not suspect that for a class of couplings we are now able to specify there is a unique event-generating algorithm. The algorithm describes the joint evolution of an individual $\mathcal{Q} \times \mathcal{C}$ system as a piecewise deterministic process. Periods of continuous deterministic evolution are interrupted by die tossing and random jumps that are accompanied by changes of state of $\mathcal{C}$-events. We call it the piecewise deterministic process algorithm, in short PDP (the term PDP has been introduced by Davis – cf. Ref. [8], and references therein). The algorithm is probabilistic which reflects the fact that the quantum world although governed by deterministic Schrödinger equation is, as we know it from experience, *open* towards the classical world of events, and the total system $\mathcal{Q} \times \mathcal{C}$ is thus open towards the future. The PDP algorithm identifies the probabilistic laws according to which times of jumps and the events themselves are chosen. Our generalized framework enables us not only to gain information about the quantum system but also to utilize it by a feedback control of the $\mathcal{Q} \times \mathcal{C}$ coupling. We can make the coupling dependent on the actual state of the classical system (which may depend on the records of previous events).

Briefly, our event enhanced formalism can be described as follows: to define an *experiment* we must start with a division $\mathcal{Q} \times \mathcal{C}$. Assuming, for simplicity, that $\mathcal{C}$ has only a finite number of states (which may be thought of as "pointer positions", but they can also represent states of a finite automaton in a quantum driven game of life) $\alpha = 1, \dots, m$, we define *event* as a *change of state* of $\mathcal{C}$. Thus there are $m^2 - m$ possible events. An experiment is then described [3] by a specific completely positive coupling V of $\mathcal{Q}$ and $\mathcal{C}$. It is specified by: (i) a family H of quantum Hamiltonians H_α parametrized by the states α of $\mathcal{C}$, (ii) a family V of $m^2 - m$ of quantum operators $g_{\alpha\beta}$, with $g_{\alpha\alpha} \equiv 0$. In our previous papers (cf. references in Ref. [3]) we have described simple general rules for constructing the $g_{\alpha\beta}$, and we described non-trivial examples, including the SQUID-tank model and generalized "cloud chamber" model that covers the GRW spontaneous localization model as a particular, homogeneous, case. The self-adjoint operators H_α determine the unitary part of quantum evolution between jumps, while $g_{\alpha\beta}$ determine jumps, their rates and their probabilities, as well as the non-unitary and non-linear contribution to the continuous evolution between jumps. As an example, in the SQUID-tank model the variable α is the flux through the coil of the classical radio-frequency oscillator circuit, and it affects, through a transformer, the SQUID Hamiltonian. $g_{\alpha\beta}$ have also a very simple meaning there [9] – they are shifts of the classical circuit momentum caused by a (smoothed out, operator-valued) quantum flux.

The time evolution of statistical states of the total $\mathcal{Q} \times \mathcal{C}$ system is described by the Liouville equation,

$$\dot{\rho}_\alpha = -\mathrm{i}\left[H_\alpha, \rho_\alpha\right] + \sum_\beta g_{\alpha\beta}\, \rho_\beta\, g^\star_{\alpha\beta} - \tfrac{1}{2}\{\Lambda_\alpha, \rho_\alpha\}, \tag{1}$$

where

$$\Lambda_\alpha = \sum_\beta g^\star_{\beta\alpha}\, g_{\beta\alpha}, \tag{2}$$

and the $\{\ ,\ \}$ stands for anti-commutator. The operators H_α and $g_{\alpha\beta}$ can be allowed to depend explicitly

[3] It is not necessary to discuss the general concept of a completely positive coupling here (the interested reader can find a discussion and references in Ref. [6]).

on time, so that the intensity of the coupling can be controlled. Moreover, to allow for phase transitions the quantum Hilbert space may change with α. One can show that the above Liouville equation determines a piecewise deterministic process (PDP) that generates histories of individual systems. Within our framework that process is *unique*. Our PDP is given by the following simple algorithm:

PDP algorithm 1. Let us assume a fixed, sufficiently small, time step $\mathrm{d}t$. Suppose that at time t the system is described by a quantum state vector ψ, $\|\psi\| = 1$ and a classical state α. Compute the scalar product $\lambda(\psi,\alpha) = \langle\psi, \Lambda_\alpha \psi\rangle$. Then toss dies and choose a uniform random number $r \in [0,1]$, and jump if $r < \lambda(\psi,\alpha)\,\mathrm{d}t$, otherwise do not jump. When jumping, toss dies and change $\alpha \to \beta$ with probability $p_{\alpha\to\beta} = \|g_{\beta\alpha}\psi\|^2/\lambda(\psi,\alpha)$, and change $\psi \to g_{\beta\alpha}\psi/\|g_{\beta\alpha}\psi\|$. If not jumping, change

$$\psi \to \frac{\exp(-\mathrm{i}H_\alpha\,\mathrm{d}t - \frac{1}{2}\Lambda_\alpha\,\mathrm{d}t)\psi}{\|\exp(-\mathrm{i}H_\alpha\,\mathrm{d}t - \frac{1}{2}\Lambda_\alpha\,\mathrm{d}t)\psi\|}, \quad t \to t + \mathrm{d}t.$$

Repeat the steps.

Remark. Another method of generating jump times is to select a random number $r \in [0,1]$ and proceed with the continuous time evolution by solving $\dot{\psi} = (-\mathrm{i}H_\alpha - \frac{1}{2}\Lambda_\alpha)\psi$ until $\|\psi\|^2 = r$ – see Ref. [10]

EEQT proposes that the PDP algorithm describes *in an exact way* all real events as they occur in Nature, provided we specify correctly $\mathcal{Q}, \mathcal{C}, H$ and V. In the following section we will formulate more precisely the basic structure of EEQT.

2. Mathematical scheme of EEQT

Let us describe the mathematical framework that we use. In order to define events, we introduce a classical system $\mathcal{C}$. Then possible events are identified with changes of a (pure) state of $\mathcal{C}$. Let us consider the simplest situation corresponding to a finite set of possible events. If necessary, we can handle infinite dimensional generalizations of this framework. The space of states of the classical system, denoted by $\mathcal{S}_\mathrm{c}$, has m states, labeled by $\alpha = 1,\dots,m$. These are the pure states of $\mathcal{C}$. They correspond to possible results of single observations of $\mathcal{C}$. Statistical states of $\mathcal{C}$ are probability measures on $\mathcal{S}_\mathrm{c}$ – in our case just sequences $p_\alpha \geqslant 0, \sum_\alpha p_\alpha = 1$. They describe ensembles of observations.

We will also need the algebra of (complex) observables of $\mathcal{C}$. This will be the algebra $\mathcal{A}_\mathrm{c}$ of complex functions on $\mathcal{S}_\mathrm{c}$ – in our case just sequences $f_\alpha, \alpha = 1,\dots,m$, of complex numbers.

It is convenient to use Hilbert space language even for the description of that simple classical system. Thus we introduce an m-dimensional Hilbert space $\mathcal{H}_\mathrm{c}$ with a fixed basis, and we realize $\mathcal{A}_\mathrm{c}$ as the algebra of diagonal matrices $F = \mathrm{diag}(f_1,\dots,f_m)$.

Statistical states of $\mathcal{C}$ are then diagonal density matrices $\mathrm{diag}(p_1,\dots,p_m)$, and pure states of $\mathcal{C}$ are vectors of the fixed basis of $\mathcal{H}_\mathrm{c}$.

Events are ordered pairs of pure states $\alpha \to \beta$, $\alpha \neq \beta$. Each event can thus be represented by an $m\times m$ matrix with 1 at the (α,β) entry, zero otherwise. There are $m^2 - m$ possible events. Statistical states are concerned with ensembles, while pure states and events concern individual systems.

The simplest classical system is a yes–no counter. It has only two distinct pure states. Its algebra of observables consists of 2×2 diagonal matrices.

We now come to the quantum system. Here we use the standard description. Let $\mathcal{Q}$ be the quantum system whose bounded observables are from the algebra $\mathcal{A}_\mathrm{q}$ of bounded operators on a Hilbert space $\mathcal{H}_\mathrm{q}$. Its pure states are unit vectors in $\mathcal{H}_\mathrm{q}$; proportional vectors describe the same quantum state. Statistical states of $\mathcal{Q}$ are given by non-negative density matrices $\hat{\rho}$, with $\mathrm{Tr}(\hat{\rho}) = 1$. Then pure states can be identified with those density matrices that are idempotent, $\hat{\rho}^2 = \hat{\rho}$, i.e. with one-dimensional orthogonal projections.

Let us now consider the total system $T = \mathcal{Q} \times \mathcal{C}$. Later on we will define "experiment" as a coupling of $\mathcal{C}$ to $\mathcal{Q}$. That coupling will take place within T. First, let us consider statistical description, only after that we shall discuss dynamics and coupling of the two systems.

For the algebra $\mathcal{A}_\mathrm{t}$ of observables of T we take the tensor product of algebras of observables of $\mathcal{Q}$ and $\mathcal{C}$: $\mathcal{A}_\mathrm{t} = \mathcal{A}_\mathrm{q} \otimes \mathcal{A}_\mathrm{c}$. It acts on the tensor product $\mathcal{H}_\mathrm{q} \otimes \mathcal{H}_\mathrm{c} = \oplus_{\alpha=1}^m \mathcal{H}_\alpha$, where $\mathcal{H}_\alpha \approx \mathcal{H}_\mathrm{q}$. Thus $\mathcal{A}_\mathrm{t}$ can be thought of as algebra of *diagonal* $m \times m$ matrices $A = (a_{\alpha\beta})$, whose entries are quantum operators: $a_{\alpha\alpha} \in$

$\mathcal{A}_q$, $a_{\alpha\beta} = 0$ for $\alpha \neq \beta$. The classical and quantum algebras are then subalgebras of $\mathcal{A}_t$; $\mathcal{A}_c$ is realized by putting $a_{\alpha\alpha} = f_\alpha I$, while $\mathcal{A}_q$ is realized by choosing $a_{\alpha\beta} = a\delta_{\alpha\beta}$. Statistical states of $\mathcal{Q} \times \mathcal{C}$ are given by $m \times m$ diagonal matrices $\rho = \mathrm{diag}(\rho_1, \dots, \rho_m)$ whose entries are positive operators on $\mathcal{H}_q$, with the normalization $\mathrm{Tr}(\rho) = \sum_\alpha \mathrm{Tr}(\rho_\alpha) = 1$. Tracing over $\mathcal{C}$ or $\mathcal{Q}$ produces the effective states of $\mathcal{Q}$ and $\mathcal{C}$ respectively: $\hat{\rho} = \sum_\alpha \rho_\alpha$, $p_\alpha = \mathrm{Tr}(\rho_\alpha)$.

Duality between observables and states is provided by the expectation value $\langle A \rangle_\rho = \sum_\alpha \mathrm{Tr}(A_\alpha \rho_\alpha)$.

We consider now dynamics. Quantum dynamics, when no information is transferred from $\mathcal{Q}$ to $\mathcal{C}$, is described by Hamiltonians H_α, that may depend on the actual state of $\mathcal{C}$ (as indicated by the index α). They may also depend explicitly on time. We will use matrix notation and write $H = \mathrm{diag}(H_\alpha)$. Now take the classical system. It is discrete here. Thus it cannot have continuous time dynamics of its own.

Now we come to the crucial point – the coupling. A *coupling* of $\mathcal{Q}$ to $\mathcal{C}$ is specified by a matrix $V = (g_{\alpha\beta})$, with $g_{\alpha\alpha} = 0$. To transfer information from $\mathcal{Q}$ to $\mathcal{C}$ we need a non-Hamiltonian term which provides a completely positive (CP) coupling. We consider couplings for which the evolution equation for observables and for states is given by the Lindblad form,

$$\dot{A} = \mathrm{i}[H, A] + \mathcal{E}\left(V^\star A V\right) - \tfrac{1}{2}\{\Lambda, A\}, \tag{3}$$

$$\dot{\rho} = -\mathrm{i}[H, \rho] + \mathcal{E}(V \rho V^\star) - \tfrac{1}{2}\{\Lambda, \rho\}, \tag{4}$$

where $\mathcal{E} : (A_{\alpha\beta}) \mapsto \mathrm{diag}(A_{\alpha\alpha})$ is the conditional expectation onto the diagonal subalgebra given by the diagonal projection, and

$$\Lambda = \mathcal{E}\left(V^\star V\right). \tag{5}$$

We can also write it down in a form not involving $\mathcal{E}$,

$$\dot{A} = \mathrm{i}[H, A] + \sum_{\alpha \neq \beta} V^\star_{[\beta\alpha]} A V_{[\beta\alpha]} - \tfrac{1}{2}\{\Lambda, A\}, \tag{6}$$

with Λ given by

$$\Lambda = \sum_{\alpha \neq \beta} V^\star_{[\beta\alpha]} V_{[\beta\alpha]}, \tag{7}$$

and where $V_{[\alpha\beta]}$ denotes the matrix that has only one non-zero entry, namely $g_{\alpha\beta}$ at the α row and β column. Expanding the matrix form we have

$$\dot{A}_\alpha = \mathrm{i}[H_\alpha, A_\alpha] + \sum_\beta g^\star_{\beta\alpha} A_\beta g_{\beta\alpha} - \tfrac{1}{2}\{\Lambda_\alpha, A_\alpha\}, \tag{8}$$

$$\dot{\rho}_\alpha = -\mathrm{i}[H_\alpha, \rho_\alpha] + \sum_\beta g_{\alpha\beta} \rho_\beta g^\star_{\alpha\beta} - \tfrac{1}{2}\{\Lambda_\alpha, \rho_\alpha\}, \tag{9}$$

where

$$\Lambda_\alpha = \sum_\beta g^\star_{\beta\alpha} g_{\beta\alpha}. \tag{10}$$

Again, the operators $g_{\alpha\beta}$ can be allowed to depend explicitly on time.

Following Ref. [11] we now define *experiment* and *measurement*:

Definition. An *experiment* is a *CP* coupling between a quantum and a classical system. One observes then the classical system and attempts to learn from it about characteristics of state and of dynamics of the quantum system.

Definition. A *measurement* is an experiment that is used for a particular purpose: for determining values, or statistical distribution of values, of given physical quantities.

Remark. The definition of experiment above is concerned with the *conditions* that define it. In the next section we will discuss the PDP algorithm that simulates a typical *run* of a given experiment. In practical situations it is rather easy to decide what constitutes $\mathcal{Q}$, what constitutes $\mathcal{C}$ and how to write down the coupling. Then, if necessary, $\mathcal{Q}$ is enlarged, and $\mathcal{C}$ is shifted towards more macroscopic and/or more classical. However, the new point of view that we propose allows us to consider our whole Universe as "experiment" and we are witnesses and participants of one particular run. Then the question arises: *what is the true $\mathcal{C}$?* This question is yet to be answered. Some hints can be found in the closing section of Ref. [3].

3. Statistical ensembles, individual systems, and the PDP algorithm

Time evolution in the standard quantum theory of closed systems is unitary reversible. In the quantum theory of open systems, dissipative, irreversible evolution is being used. But there it is considered only as an approximate description, not the exact one. It is useful when external unknown factors disturb the true unitary dynamics, and we either do not need, or are not able because of computational complexity, to use the exact unitary dynamics. The main difference between unitary reversible and dissipative irreversible evolutions is in their mixing properties. Unitary evolution maps pure states into pure states, while dissipative evolution maps pure states into mixtures. Pure states describe individual systems. Mixtures describe statistical ensembles. Thus when evolution preserves purity of states, then we may assume that it concerns individual systems. Things change when we want to move from mere lasting to events that happen in time. From continuous and deterministic evolution of possibilities to discrete realization of actualities, when God allows us either to rely on chance or to choose. Standard quantum theory is helpless when it comes to generation of events. But the material world around us, the living nature, the phenomena that we want to understand – all that – we perceive only through events, and nothing but events. Thus standard quantum theory must be enhanced. The only way to make a quantum system to be coupled to a classical event-carrying system is via dissipative dynamics as described in the previous section. But the Liouville equation with a nontrivial coupling term must lead from pure states to mixed states. Thus it does not describe individual systems – it describes statistical ensembles. What describes individual systems is the PDP algorithm as given at the end of Section 1.1. A priori one could think that there may be many such algorithms with the property that, after averaging over individual sample paths, reproduce a given statistical behaviour. Here that is not so. We have shown that the PDP algorithm is unique. The proof is given in an infinitesimal form in Ref. [3]. A rigorous global proof can be found in Ref. [12]. This fact, i.e. uniqueness of the random process that reproduces the master equation, distinguishes PDP from the quantum Monte Carlo methods used in quantum optics. We discuss this fact in some detail in Ref. [3].

The PDP algorithm is the most important new result of our approach. It is simple, it is universal, it is useful. We have already mentioned it in the introduction that all the standard postulates of quantum theory about measurements and their probabilities can be deduced from the PDP via suitable couplings. We have discussed this subject elsewhere in the aforementioned references. In particular we succeeded in reproducing real time formation of particle tracks and interference patterns [13] [4]. We are investigating new applications of the algorithm. But for successful applications in new situations we need one more piece in the theory – a piece that is still missing. We know how to describe *measurements*, but we must also know how to describe state preparations. In principle state preparation can be thought of as a measurement with sample selection, so it could essentially fit into the scheme that we have already described. However, we need more. We need to learn how to describe preparation of multiparticle states that look like individual particle states at each given time. Only then we will be able to make a realistic simulation of experiments in neutron interferometry or electron holography, when a source produces weak but coherent particle beams. Work in this direction is in progress.

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