Event Enhanced and Piecewise Deterministic Quantum Theory or the Right Jump at the Right Place

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Abstract: The purpose of Event Enhanced Quantum Theory (EEQT) is to allow for a precise meaning to the concepts of "event", "experiment" and "measurement". Within EEQT one obtains not only Liouville equations describing the continuous dynamics of statistical ensembles but also a unique minimal piecewise deterministic random Markov process (PDP) than can be used for computer simulations of real time series for experiments on individual quantum systems. EEQT is therefore particularly relevant to today's experimental Quantum Physics since new technology needs new laws and its range of applications is rather wide. As an example a cloud chamber model will be discussed. In a particular, homogeneous, case this model contains GRW spontaneous localization model. All probabilistic interpretation of Standard Quantum Theory can be derived from the formalism of EEQT. Moreover EEQT has no need for observers or minds. EEQT is a precise and predictive theory not only giving enhanced answers but also inviting asking new questions for example on the grand vision of a Quantum Theory of history à la Gell Mann–Hartle or on Connes' version of the Standard Model. In conclusion EEQT is a minimal extension of the Standard Quantum Theory that accounts for events and satisfies the needs of human experience and modern technology.

1 Greatness and Troubles with Orthodox Quantum Theory

To start at the beginning we have first to say that Orthodox Quantum Theory (OQT) has proved to be incredibly powerful, practical and successful in the description of the properties of atoms, molecules and elementary particles. There seems to be no limit to the versatility of the Schrödinger equation. OQT was and is outstandingly successful in *computing* not only those phenomena for which it was invented but also numerous others in Physics, Chemistry and Biology making therefore wonderful advance in technology possible. On the other side the interpretation of OQT is still today the most controversial problem in the foundations of physics. Its successes resemble those of the Ptolemeian System

– which was also no doubt successful. It was John Bell's point of view that no formulation of OQT was free of fatal flaws [1, 2]. N. Bohr was persuaded from the consistency of OQT. For A. Einstein OQT does not allow a complete description of Nature and was only the first step of the final theory. "Red black magic calculus" is how he described OQT in a letter in 1925. "The fact that an adequate representation of Quantum Mechanics has been so long delayed is no doubt caused by the fact that Niels Bohr brainwashed a whole generation of theorists into thinking that the job was done fifty years ago" wrote Murray Gell Mann 1979. OQT considers two types of incompatible time evolution U and R, U denoting the unitary evolution resulting from Schrödinger's equation and R the reduction of the quantum state. U is linear, deterministic, local, continuous and time reversal invariant, while R is probabilistic, non–linear, discontinuous and acausal. For a fundamental physical theory this situation is not very satisfactory but J. Bell claimed that OQT works for all practical purposes (FAPP). Two options are possible for completing OQT. According to John Bell [2] "Either the wave function is not everything or it is not right ...". Gisin and Percival formulated the thesis that "the Schrödinger equation is no longer the best for all practical purposes" [3].

As emphasized already by E. Schrödinger [4] completely missing in OQT is an explanation for experimental facts i.e. a description of the actual individual times series of events. What is needed is a consistent framework which can be used to retrieve the existence of events despite the probabilistic character of quantum physics. A theory allowing the description of single systems is today necessary since advances in technology make fundamental experiments on individual quantum systems possible. Events can be recorded and have definitely no place in OQT. The importance of the events have been stressed by several authors. H.P. Stapp emphasized the role of events in the world process [5–7]. R. Haag [8] drawed attention to the fact that "an event in quantum physics is discrete and irreversible" and that "we must assume that the arrow of time is encoded in the fundamental laws ...". He also suggested [9] that "transformation of possibilities into facts must be an essential ingredient which must be included in the fundamental formulation of the theory". Each event must have in any case three characteristic properties:

– it is classical
– it is discrete
– it is irreversible .

For us the adjective "classical" has a well defined meaning: To each particular experimental situation corresponds a minimal set of events revealing us the Heisenberg transition from the possible to the actual and these events which can be recorded obey the rules of classical logic of Aristotle and Boole. Indeed an event must obey to the classical "yes–no" logic; to be an event it must never be in a superposition of being happened and being unhappened. Since an event must happen wholly it is necessary discrete. Finally each event is irreversible because it must have left a trace. Even if this trace can be erased, the very act of erasing will change the future, not the past. We believe that the events, and nothing but events are pushing forward the arrow of time. Irreversible laws are fundamental and reversibility is only an approximation. This fact has been recently once more emphasized by H. Rauch [10] referring to a work of H.A. Lorentz [11]. A typical event is for instance a track in a bubble chamber making elementary particles visible, or the click of a detector. Once the three characteristics of an event are accepted – and they are completely evident for any experimenter – it becomes clear what is necessary if we want to enhance OQT so as to include events into it.

First we must allow the formalism to include from the beginning classical quantities. Indeed we believe that is better when this assumption is done openly rather than indirectly as it is almost ever the case. Second we must take into account an irreversible coupling between classical and quantum degrees of freedom. Let us stress that in our opinion this minimal irreversibility is not the manifestation of noise, chaos or environment but expresses simply the universal fact that information must be paid with dissipation. Finally we need an important third step, namely to learn how to describe finite time series of events from which expectation values can be computed. Moreover, as human beings, we want not only to be able to compute statistical properties of ensembles but also to be in position to simulate finite time series for individual systems since we cannot enter twice the same place in the same stream of time.

In the following section we will describe in a condensed form what seems to us to be the minimal enhancement of quantum theory completing the needs of human experience and modern technology. In [12–18] we proposed mathematical and physical rules to describe

- the two kinds of time evolution of quantum systems namely continuous and stochastic
- the flow of information from the quantum system to the classical event-space
- the control of quantum states and processes by classical parameters.

2 All You Have in Mind to Know About EEQT

It is one of the aims of this section to express in a condensed form and partially informal way the philosophical backbone that can be extracted from the several models discussed in [12–20]. In EEQT the quantum system Q is coupled to a "classical" space C, where events do happen. We consider the total system $\Sigma = Q \times C$. Let us denote by X_c the classical event–manifold and by $\mathcal{H}_q$ the Hilbert space associated to the quantum system Q. A classical pure state is nothing else as a point in X_c and the coordinates of this point correspond exactly to the properties of C.

From the structural and from the mathematical point of view, the three most essential features of EEQT are

- tensoring of the non commutative quantum algebra of observables $\mathcal{A}_q = \mathcal{L}(\mathcal{H}_q)$ with a commutative algebra of continuous functions $\mathcal{A}_C = C(X_C)$.

We consider therefore the behaviour associated to the total algebra of observables $\mathcal{A}_t = \mathcal{L}(\mathcal{H}_q) \otimes C(X_C)$.

- replacing Schrödinger's unitary dynamics of pure states with a suitable completely positive semigroup $\alpha_t = e^{tL}$ describing the time evolution of ensembles. Time evolution of ensembles of coupled systems, prepared by the same algorithm, is described by a Liouville equation in the Hilbert space $\mathcal{H}_t$ of the total system with $\mathcal{H}_t = \mathcal{H}_q \otimes L^2(X_C)$. The main characteristic of the dynamics is that it does not map pure states into pure states but it is well defined on the level of the density matrices where it preserves convexity, positivity and trace.
- interpreting the continuous time evolution of the statistical states of the total system Σ in terms of a PDP, Markov process taking values in the set of pure states $S_1(\mathcal{H}_q) \otimes X_C$ of the total system with $S_1(\mathcal{H}_q) = \{\psi \in \mathcal{H}_q \mid \|\psi\| = 1\}$. The process consists of pairs (quantum jump, classical event) interrupting random periods of Schrödinger–type continuous in general non–linear dynamics. Time evolution of the PDP is derived from the Liouville equation. At random times distributed according to a specific inhomogeneous Poisson process jumps occur.

There are jumps of the quantum state vectors and also at the same time jumps of the states of C. But we do not observe Hilbert space vectors. Are quantum jumps "real"? They are "real" but belong to the "implicate order". Events are real and belong to "explicate order". Indeed the classical jumps we can see (to measure a quantity we must look at it) and these classical events can be recorded if necessary. Knowing this PDP one can answer many (perhaps even all) kinds of questions about time correlations of events as well as simulate numerically the possible histories of individual systems. Within EEQT there need not be cat paradoxes anymore – cats are allowed to behave as cats; we cannot predict individual events as they are random, but we can simulate the observations of individual systems. The pure quantum states can be viewed as a hidden variable and EEQT as a purely classical theory accounting for quantum phenomena. In EEQT the word measurement instead of being banned as suggested by J. Bell can be given now a precise and acceptable meaning. An *experiment* is a completely positive (CP) coupling between a quantum system and a classical event space. One observes then the classical system C and attempts to learn from it about characteristic of state and of dynamics of the quantum system Q. A *measurement* is an experiment that is used for a particular purpose namely for determining values, or statistical distribution of values of given physical quantities. The aim of any well performed measurement is therefore to get a maximum of information on Q and to pay for it with a minimum of dissipation.

By partial tracing each state ρ of the total system projects onto an effective quantum state $\hat{\rho} \equiv \Pi_q(\rho)$ and an effective classical $\mu \equiv \Pi_C(\rho)$.

Let us consider dynamics. The time evolution of the total system is given by a semigroup $\phi_t = e^{tL}$ of CP maps of $\mathcal{A}_{tot}$ where L is a Lindblad generator (cf. [18]). There is a simple method of constructing appropriate couplings. Suppose that we consider quantum properties F_α , $\alpha = 1, 2, \ldots$ that we want to measure. For each α we try to find a transformation of X_C with the following interpretation: If the quantum system Q has property F_α while the classical–event system is in a state x then C switches from x to a new state $\alpha(x)$. Denoting $\rho(t) = \phi_t(\rho_0)$ the time evolution of the states is given by the Liouville equation

$$\dot{\rho}_x(t) = -i[H_x, \rho_x(t)] + \sum_\alpha V_\alpha \rho_{\alpha(x)}(t) V_\alpha^* \\ -\frac{1}{2}\{\Lambda, \rho_x\}$$

where we have denoted

$$\Lambda = \sum V_\alpha^* V_\alpha \ .$$

The Liouville equation can be therefore written as a sum

$$\dot{\rho}(t) = -i[H \ , \rho(t)] + (\dot{\rho}(t))_{irr} \ .$$

The first term $-i[H_x, \rho_x(t)]$, diagonal in x, contains all deterministic parts of the time evolution, while the second accounts for irreversible processes associated to the coupling. It is important to notice that if the quantum Hamiltonian does not depend on the state of the classical system i.e. $H_x \equiv H$ for each $x \in X_C$, and if the maps α of X_C are one to one and onto, then the Liouville equation can be summed up over x which implies that the time evolution for the effective quantum states $\hat{\rho}$ obtained by partial tracing separates and we have

$$\dot{\hat{\rho}}(t) = -i[H, \hat{\rho}(t)] + \sum_\alpha V_\alpha \hat{\rho}(t) V_\alpha^* - \frac{1}{2}\{\Lambda, \hat{\rho}\}.$$

It should be stressed that this separating property of the Liouville equation describing the dynamics of the total system $\Sigma = Q \times C$ need in general not to hold. That is what happens in the SQUID–tank model [14].

<u>Remark</u> Assuming, for simplicity, that C has only finite number of states (which may be viewed as "pointer positions") $x = 1, \ldots, m$, an event is a change of state of C. Thus there are $m^2 - m$ possible events. An experiment is then described by a family H of quantum Hamiltonians and a family of $m^2 - m$ quantum operators g_{xy} with $g_{xx} = 0$.

A complete general theory of dissipative couplings of quantum systems to classical ones, does not yet exist. The best we can do is to study a lot of examples. In the following section we will sketch a characteristic situation describing a model for a cloud chamber. For every example we have considered, a PDP has been constructed that takes place on the space of pure states of the total system and which reproduces the Liouville equation by averaging. The theory of PDP is described in a recent book by M.H. Davis [21]. A PDP is determined by its local characteristics namely

i) A vector field which determines a flow on the state space

ii) A jump rate function λ
iii) A transition probability matrix Q.

Each observable A of the total system defines now a function $f_A(\psi, x)$ on the space $S = \{(\psi, x)\}$ of pure states of the total system. It can be shown that the time evolution $\frac{d}{dt} f_A(\psi, X)$ for observable can be written in a Davis form.

Let us now describe the PDP on pure states of the total system $\Sigma = Q \times C$ that leads to the Liouville equation after averaging over paths. For a derivation we refer to [20]. Suppose at initial time $t = 0$ the quantum system is in the pure state $\psi_0 \in \mathcal{H}_q$ and the classical system is in the pure state $\delta_{x_0} \approx X_0$. Then the time evolution of the quantum state is given by

$$\psi(t) = \frac{e^{-iH_{x_0}t - \frac{\Lambda}{2}t}}{\left\| e^{-iH_{x_0}t - \frac{\Lambda}{2}t} \right\|}$$

while the classical system remains at x_0 until a jump occurs at some random time t_1. The random jump time t_1 is governed by a nonhomogeneous Poisson process for which the probability $p(t, t + \Delta t)$ for the jump to occur in the time interval $(t, t + \Delta t)$, provided it did not occurred yet, is given by the formula

$$p(t, t + \Delta t) = 1 - e^{-\int_t^{t+\Delta t}} \lambda(\psi(s))ds$$

where

$$\lambda(\psi) \equiv < \psi, \Lambda\psi > \quad .$$

When the jump occurs at $t = t_1$ then the classical system C jumps from x_0 to $\alpha_1(x_0)$ while quantum state vector jumps at the same time from its actual value $\psi(t_1)$ to $\psi_1 = V\alpha_1\psi(t_1)/ \parallel V\alpha_1\psi(t_1) \parallel$ and the process starts. The probability p_α of choosing a particular value of α is given by

$$p_\alpha = \frac{\parallel V_\alpha \psi(t_1) \parallel^2}{\lambda(\psi(t_1))} \quad .$$

A fully satisfactory mathematical justification of the uniqueness of the minimal PDP starting from the Liouville equation can be given. The uniqueness of the PDP follows from the special form of the Liouville equation in EEQT. It describes transfer of information from Q to C without introducing unnecessary dissipation reflected by the fact that there should be zeros on the diagonal of the coupling V–matrix. Starting with a pure state (ψ, x) of the total system after time dt we obtain a mixed state; there will be mixing along classical, which is uniquely decomposable and mixing along quantum which is non uniquely decomposable. But it can be shown that mixing along classical is of the order dt and on the other hand mixing along quantum is only of the order $(dt)^2$ which implies infinitesimally unicity [22]. For a mathematically rigorous global proof see [23]. In other words our dissipation does not result from quantum noise but is nothing else as the necessary minimal price to pay for any bit of information received from the quantum system.

3 Cloud Chamber Model

As an illustration of EEQT we will now sketch the description of a non–relativistic cloud chamber model. For more details we refer to [19,20]. The points $a \in E_d \subset \mathbf{R}^3$ will parametrize detectors and finite sets of points will play the role of $x-s$ of the previous section. For each $a \in E_d$, in other words for each detector, let there be given a function $f_a(x)$ on E_d. Physically $f_a(x)$ describes the sensitivity of the detector located at the point a. For instance we could take typically for $f_a(x)$ a Gaussian function

$$f_a(x) = \lambda^{1/2} \left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha(x-a)^2} \; .$$

The height would then be approximately inversely proportional to the square root of the response time of the local counter while the width to its spatial extension. A point limit will then correspond to the limit $f_a^2(x) \rightarrow \lambda\delta(x-a)$, where δ denotes the Dirac measure in a. Our classical system is a continuous medium of 2–states detectors. At each point a the medium is one of two states: "on" represented by $\binom{1}{0}$ or "off" represented by $\binom{0}{1}$. The space of classical events can be identified with the space of finite subsets of E_d from which it follows that the total system is described by families $\{\rho_\Gamma\}_{\Gamma \subset E}, \Gamma$ finite subset of E_d, such that $\sum_\Gamma Tr\rho_\Gamma = 1$. What remains to be specified to define our model is the transformation $\alpha : \Gamma \rightarrow \alpha(\Gamma)$ but there is a natural choice namely each α flips the detector state at $x = a$. We can write it also as $\alpha(\Gamma) = \{a\}\Delta\Gamma$, where Δ denotes the symmetric difference.

The quantum mechanical Hilbert space is $\mathcal{H}_q = L^2(E_d, dx)$. The Liouville equation governing the time evolution of the total system is given by

$$\dot{\rho}_\Gamma = -i[H_\Gamma, \rho_\Gamma] + (\dot{\rho}_\Gamma)_{irr}$$

where $(\dot{\rho}_\Gamma)_{irr}$ is given by a Lindblad generator

$$\begin{aligned}(\dot{\rho}_\Gamma)_{irr} &= L_{int}(\rho) \\ &= \int_{E_d} da f_a \rho_{\Gamma\Delta\{a\}} f_a - \frac{1}{2}\{\Lambda, \rho_\Gamma\}\end{aligned}$$

with

$$\Lambda(x) = \int_{Ed} f_a^2(x) dx \; .$$

We can also construct the associated minimal PDP. We obtain a Davis generator corresponding to rate function $\lambda(\psi) =< \psi, \Lambda\psi >$ and probability kernel with non–zero elements given by

$$Q(\psi, \Gamma; d\psi', \alpha(\Gamma)) = \frac{\| f_a\psi \|^2}{\lambda(\psi)} \delta \left(\psi' - \frac{f_a\psi}{\| f_a\psi \|}\right) d\psi' \; .$$

Time evolution between jumps is given by

$$\psi_t = \frac{e^{-iHt - \frac{\Lambda t}{2}}\psi_0}{\left\| e^{-iHt - \frac{\Lambda t}{2}}\psi_0 \right\|} .$$

The jump consists of a pair (classical event, quantum jump). The classical medium jump at a with probability density $p(a, \psi_{t_1}) = \| f_a \psi_{t_1} \|^2 / \lambda(\psi_{t_1})$ (flip of the detector) while the quantum part of the jump is jump of the Hilbert space vector ψ_{t_1} to $f_a \psi_{t_1} / \| f_a \psi_{t_1} \|$ and the process starts again. We recognize the von Neumann–Lüders projection postulate. The random jump time t_1 is governed by the inhomogeneous Poisson process with intensity $\lambda(\psi_{t_1})$. the probability for the jump to take place at place a is given by

$$p_a = \frac{\| f_a \psi_t \|^2}{\int \| f_a \psi_t \|^2 \, da} .$$

For $f_a^2(x) \to \delta(x - a)$ we recover $p_a = | \psi_t(a) |^2$ i.e. the Born interpretation of the wave function.

In [19,20] we discuss the case of a passive homogeneous medium for which H does not depend on the actual state of the medium. This implies that the reduced quantum state separates. For constant jump rate we recover the spontaneous localization model à la GRW [24].

4 Summary and Conclusions

Our models of coupling quantum systems to classical event–spaces can be surely criticized as being too phenomenological. Nevertheless EEQT is a minimal extension of OQT satisfying the need of human experience and modern technology and supplying the interface between the quantum world and the events. The cloud chamber model and EEQT do not involve observers and minds. What EEQT needs is computing power and an effective random number generator. Let us in this context formulate an open fundamental question: Can random numbers generators be avoided and replaced by deterministic algorithms of simple and clear meaning?

The fact that our models avoiding the concepts of observers and minds are totally objective does not imply that we do not appreciate the importance of the mind–body problem. In our opinion understanding the problem of mind needs also Quantum Theory and perhaps even more. Our feeling is that models in the spirit of EEQT can have applications in biology. Living organisms are coherent open systems with a program dependent of molecular recording processes. Variations take place there on the quantum level and are translated and amplified to generate macroscopic variations ....

The second important advantage of EEQT is that it provides as sketched in Section 2 its own interpretation. Moreover EEQT allows the description of single systems via PDP and provides in this way very effective methods for numerical simulation of experimental events.

After suitable transformation we believe that EEQT can be acceptable even by quantum purists who claim the universality of quantum theory and do not recognize the fact that there are classical events. They may consider our C as a "preferred basis" and then notice that the completely positive semi-group describing the dynamics always respect this basis. They will also appreciate the fact that only in special situations the effective evolution of the reduced quantum state $\hat{\rho} \equiv \Pi_1(\rho)$ separates but that this separation is not at all necessary in EEQT. In [12, 13, 15–20] we discuss following experiments and topics in the EEQT framework: Measurement-like processes, Stern–Gerlach experiment, Quantum Zeno Effect, EPR, SQUID–tank interaction, efficiency versus accuracy by measurement, simultaneous measurement of non-commuting observables, meaning of the wave function. In EEQT we need only to postulate that events can be observed. All the rest can be derived from this assumptions. All quantum mechanical probabilistic interpretation can be derived from the formalism of EEQT. EEQT invites also to ask new questions since we are tempted to consider the PDP as a "world process" to the entire universe including all kinds of "observers". The questions to be asked now are: what is time? What is classical? What is V describing the binamics? Of course we cannot provide answers but we can provide hints [22].

To draw a conclusion let us say that provided EEQT correctly accounts for experimental results it offers some new ways of seeing things and new mathematics providing an additional perspective on the duality between the potential and the actual, statistical ensembles and individual systems, waves and particles and the deterministic and the random. In his 1933 Spencer lecture A. Einstein mentioned the success not only of classical mechanics but also of the statistical interpretation of quantum theory. He added "I still believe in the possibility of giving a model of reality which shall represent events themselves and not merely the probability of their occurrence". EEQT can be considered as a step in this direction.

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