Time of Arrival in Event Enhance

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Abstract

The new solution to the problem of time of arrival in
herein. It allows for computer simulation of particle c
interpretation. It also suggests new experiments that ca
quantum particle detect a detector without being detecte

1 Introduction

One of the most troublesome deficiencies of Textbook
questions about timing of experimental events unanswer
deficiency is that an *experimental event* (or *measureme*
standard theory [1,2]. Due to this deficit, Bell felt the
quantum theory [3].

Recently we have developed a semi-phenomenologica
and also has a predictive power that is stronger than the
is why it was entitled *Event Enhanced Quantum Theory*

EEQT can be thought of as a formalism implementing
any experimental event is classical in nature - necessarily
our colleagues what we did and what result we obtained.
from the classical is also known as the Heisenberg cut. E
theory because it considers the exact placement of the cu
and references therein) believes that the only events that a
events. Pushing the borderline between quantum and cl
interface would make EEQT into a fundamental theory
interface between matter and mind is identified. For most
borderline can be placed simply between the quantum o
apparatus or its part (e.g. its display, or pointer). EEQT
work to describe the interface and the reciprocal coupling

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cal measuring device and with the unavoidable back action

at EEQT provides the algorithm that enables us to model nces, including the *timing* of events. A discussion of other e in quantum theory has been given by Haag [6]. Here we ion in the context of *time of arrival*.

ition

question of time of arrival can be discussed is that of a e ask at what time the particle will arrive at some specific nswer this question experimentally we would set a particle me interval t between the moment the particle is released y the detector. Experiments suggest t is a random variable. nany times, and assuming the particle is always being pre- $|\psi>$, we arrive at an experimental probability distribution e probability that the particle is detected up to time t, thus

00% efficient detectors, so we have the probability $P(\infty)$ particle at all, is less than one. [1]. The standard quantum any formula for $p(t)$.
0 of Ref. [7]) has assumed, completely ad hoc, that the

$$p(t) = const|\psi(a,t)|^2, \tag{1}$$

chrödinger equation.[2] One needs, to this end, to go beyond e not so many options - one can try Nelson's Stochastic s or EEQT. The formula for time of arrival can then serve udge which of the alternatives better fits the experimental

l follow [8] and describe the formula for time of arrival ollowing Wigner, we will consider first a somewhat more time of arrival *at a given state* $|u>$. In EEQT noiseless a classical yes-no device is described by a positive oper- $= \sqrt{\kappa}|u><u|$, where κ is a phenomenological coupling imension t^{-1}. The Master Equation describing continuous s of the quantum system coupled to the detector reads:

$$= \; -\frac{i}{\hbar}[H_0, \rho_0(t)] + F\rho_1 F$$

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sion let us mention at this place that numerical simulations using our e detector suggests $P(\infty) < 0.73$

s it leads, for a Gaussian wave packet, to $P(\infty) = \infty$. Later on we q.(4)) involves integral transform of $\psi(a,t)$.

$$\dot{\rho}_1(t) \;=\; -\frac{i}{\hbar}[H_1, \rho_1] - \frac{1}{2}\{F^2$$

Suppose at $t = 0$ the detector is off, that is in the state den
is $|\psi>$, with $<\psi|\psi>= 1$. Then, according to EEQT (c
detection, that is of a change of state of the detector, durin
$1- <\psi|K(t)^{\star}K(t)|\psi>$, where

$$K(t) = \exp(-\frac{i}{\hbar}Ht - \frac{F^2}{2}t$$

It then follows that the probability $p(t)dt$ that the detector
interval (t,t+dt), provided it was not triggered yet, is given

$$p(t) = \frac{d}{dt}P(t) = \kappa \mid <u|K(t)|\psi$$

The difference between the above and Wigner's formula i
stant κ as well as the damping term $F^2/2$ in the definition
damping term together with the coupling constant that assu
the formula as in [7].
To compute $p(t)$ let us note that $p(t)$ is equal to $|\phi(t)|^2$, wh
is given by $<u|K(t)|\psi>$. Denoting by $\tilde{\phi}(z)$ the Laplace t
(cf [8]):

$$\tilde{\phi} = \frac{\sqrt{\kappa} <u|\tilde{K}_0|\psi>}{1 + \frac{\kappa}{2} \; <u|\tilde{K}_0|u>}$$

where

$$K_0(t) = \exp(-\frac{i}{\hbar}Ht).$$

This is our final formula for the Laplace transform of the
arrival.

Let us consider a free Schrödinger particle on a line,
improper position eigenstate at a, that is $<x|u>= \delta(x -$

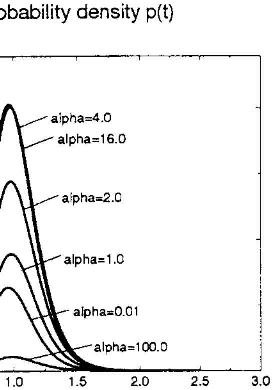
$$<u|\tilde{K}_0|u>= \tilde{K}_0(a, a; z) = \left(\frac{\hbar}{2}\right.$$

Let us denote:

$$\tilde{G}(z) = \frac{1}{1 + \frac{\kappa}{2} \; <u|\tilde{K}_0|u>} = $$

where $\epsilon = \frac{\kappa}{2}\left(\frac{\hbar m}{2i}\right)^{\frac{1}{2}}$. It can be now checked that the inv
$\tilde{G}(z)$ is given by

$$G(t) = \delta(t) + \frac{d}{dt}f(t),$$



time of arrival for a point counter placed at $a = 0$, dimen-
$m\eta\kappa/\hbar$. The incoming Gaussian wave packet of width η
elocity $v = 2\hbar/m\eta$

$$f(t) = e^{\epsilon^2 t}\mathrm{Erfc}\left(\epsilon t^{\frac{1}{2}}\right). \qquad (10)$$

n:

$$\left(\psi_0(t,a) + \int_0^t \dot{f}(s)\psi_0(t-s,a)ds\right), \qquad (11)$$

evolving wave function. The second term in the formula
on to the Wigner formula (1).
mit of infinite coupling constant. Numerical simulations
ve packet there is an optimal value of the coupling constant
iency of the detector. Increasing κ over this optimal value
of reflection of the particle by the detector. In the limit of
$P(\infty)$ drops to zero - cf. Fig 1. One may ask what is the
y $P(\infty)$ for a point counter? We do not know the answer
at the maximum efficiency is reached for a Gaussian wave
ver the detector with zero velocity. Numerical simulations
e wave packet slowly spreads out being at the same time
Figure 2 shows the dependence of the detector efficiency
s coupling constant $\alpha = m\eta\kappa/\hbar$. The maximum is attained
turns out to be ≈ 1.73. It would be desirable to have an
ur conjecture. The fact that the maximal detector efficiency
an artefact of the singular character of a pointlike detector.

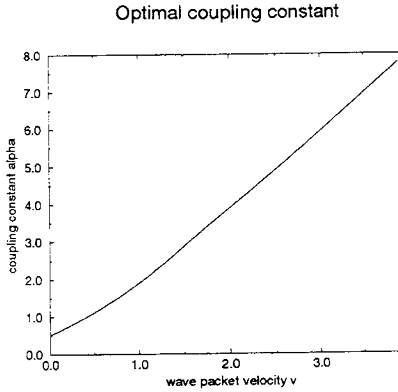


Figure 2: Optimal coupling constant as a function of veloc
The dependence pretty soon saturates to a linear one. At t

It may, however, also have some deeper meaning. If so, t
to us.

3 Conclusions

We have seen that the formula for time of arrival of a
phenomenological parameter κ characterizing the stren
particle and the sink. If κ is too small then most of the
undetected. If κ is too big, then the sink will also act
incoming wave packet there is an optimal value of the cou
efficiency.

Our formula for the time of arrival can be used to
of time–energy uncertainty relation. However, it must b
will be much more difficult than that in the original Wi
formula (11) contains an extra term which is absent in t
of a general time of arrival at a state $|u>$ Wigner's form
(5b) of Ref. [7] is also incorrect as his "or" between Eq.
a general $|u>$.

From the probabilistic point of view the process of
given state $|u>$ is an inhomogeneous Poisson process w

$$\lambda(t) = \kappa \frac{<\psi|K(t)^{\star}|u><u|K}{<\psi|K(t)^{\star}K(t)|}$$

A more general algorithm for a piecewise deterministic p

ement can be found in Ref. [4].
amping term in the propagator $K(t)$ (cf. Eq. (3) is to be
ifiable. That is, the very presence of a detector, even if
tected, changes the time evolution of the wave packet by
Hamiltonian. The phenomenon here is of the same kind
Elitzur et al. [10] and Kwiat et al. in [11]. We can say
ctor without being detected itself. Our formula for $K(t)$
tive way.
ld be interesting to obtain a relativistic version of the time
principle done by exploiting the ideas given in Ref. [12].

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