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PARTICLES MOTION IN A MULTIDIMENSIONAL UNIVERSE AND NONLINEAR HIGGS CHARGES*

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Summary

Generalized Kerner-Wong equations are derived and compared with geodesic motion of freely falling particles in a multidimensional Universe with homogeneous fibres. It is argued that the new charges which couple to the generalized Jordan-Thierry fields are nonlinear.

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Recently a dimensional reduction scheme was proposed based on a multidimensional gravitational field constrained by an imposed global symmetry group. In the present note we use the notation and the results of [1]. E is a multidimensional Universe endowed with a gravitational field g_{AB} satisfying the Killing equations

$$L_{\varepsilon_i} g_{AB} = 0 ,$$

where ε_i ,

$$[\varepsilon_i , \varepsilon_j] = C^k_{ij} \varepsilon_k$$

are basic vectors in the Lie algebra G . The reduction theorem given in [1] shows that the remaining degrees of freedom in g_{AB} can be described in terms of a gravitational field $g_{\mu\nu}(x)$ on space-time M (the manifold of G-orbits), gauge field $A^{\hat{a}}_{\mu}(x)$ on M with gauge group K = N/H, where H is the isotropy group and N is the normalizer of H in G, and Higgs fields $g_{\alpha\beta}(x) = (g_{\hat{a}\hat{b}}, g_{ab})$. The indices we use run as follows

μ, ν, σ - space-time

i, j, k - G,

α, β, γ - $S \cong G/H$,

$\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ - H ,

a, b, c - $L \cong G/N$,

$\hat{a},\hat{b},\hat{c}$ - $K \cong N/H$.

Our aim is to investigate motion of test particles in external fields $g_{\mu\nu}$, $A^{\hat{a}}_{\mu}$, $g_{\alpha\beta}$. The simplest method to get equations of motion is by the Souriau method i.e. from the invariance principles (see [2], also [3] and references there). With the notation of [3] we get:

for a vertical invariant vector field $X = (\chi^{\hat{a}})$ on P

$$(L_X\omega^{\hat{a}})_\mu = D_\mu\chi^{\hat{a}} ,$$
$$L_X\, g_{\alpha\beta} = \chi^{\hat{a}}(C_{\hat{a}\alpha,\beta} + C_{\hat{a}\beta,\alpha}) , \tag{1}$$

for a horizontal invariant vector field $X = \lambda\zeta$ (λ-horizontal lift)

$$(L_X\omega^{\hat{a}})_\mu = F^{\hat{a}}_{\mu\nu}\zeta^\nu ,$$
$$L_X\, g_{\mu\nu} = \nabla_\mu\zeta_\nu + \nabla_\nu\zeta_\mu \tag{2}$$
$$L_X\, g_{\alpha\beta} = \zeta^\mu D_\mu g_{\alpha\beta} .$$

The responce functional F can be written as

$$\langle F,\delta\kappa\rangle = \int (\tfrac{1}{2}T^{\mu\nu}\delta g_{\mu\nu} + J^\mu_{\hat{a}}\delta\omega^{\hat{a}}_\mu + \tfrac{1}{2}Z^{\alpha\beta}\delta\, g_{\alpha\beta})dt , \tag{3}$$

and assuming its vanishing for displacements $\delta\kappa$ given by (1) and (2) one gets $T^{\mu\nu} = m\dot{x}^\mu\dot{x}^\nu$, $J^\mu_{\hat{a}} = q_{\hat{a}}\dot{x}^\mu$ and evolution equations

$$\frac{D\dot{x}_\nu}{dt} = (q_{\hat{a}}/m)F^{\hat{a}}_{\mu\nu}\dot{x}^\mu + \frac{1}{2m} Z^{\alpha\beta}D_\nu g_{\alpha\beta} , \tag{4}$$

$$\frac{Dq_{\hat{a}}}{dt} = \tfrac{1}{2} Z^{\alpha\beta}(C_{\hat{a}\alpha,\beta} + C_{\hat{a}\beta,\alpha}) , \tag{5}$$

where

$$C_{\hat{a}\alpha,\beta} = g_{\beta\gamma}\, C^\gamma_{\hat{a}\alpha} . \tag{6}$$

It is remarkable that the restriction of the invariance group to K, as required by the above method, leaves completely undetermined time evolution of the "Higgs charges" $Z^{\alpha\beta}$ which couple to the Higgs fields $g_{\alpha\beta}$.

In order to get a deterministic system of equations of motion one can try to project geodesic motions in E onto M. Here, however, one meets the following difficulty: a geodesic γ in E need not be contained in the principal bundle P realized as a submanifold of E. In fact, because the projections of $\dot{\gamma}$ onto the Killing vectors are constants of motion, the angle between γ and P (or rather translations of P) remains constant. To remedy this difficulty one could use action of the global symmetry group G, to transform $\dot{\gamma}$ back to P. Then, however, another problem arises-the group element implementing the transformation is unique only modulo the stability group H of P.

As a result the effective charge looses its linearity - it takes values in the manifold of Ad(H)-orbits in $S = G/H$.
The detailed analysis and interpretation of the resulting evolution equations is difficult and will be given elsewhere [4]. Here we compare the explicit form of geodesic equations in E with (4) and (5) in order to get some idea about the nature of the Higgsonic charges $Z^{\alpha\beta}$.

The local moving frame in E (vielbein) e_A we use consists of invariant horizontal vector fields e_μ , and fundamental vertical vector fields e_α . The Christoffel symbols of g_{AB} calculated at p P are

$$\Gamma_{\alpha\beta,\gamma} = \tfrac{1}{2}(C_{\alpha\beta,\gamma} - C_{\gamma\alpha,\beta} + C_{\beta\gamma,\alpha}) , \qquad (7)$$

$$\Gamma_{\alpha\mu,\beta} = \Gamma_{\mu\alpha,\beta} = - \Gamma_{\alpha\beta,\mu} = \tfrac{1}{2} D_\mu g_{\alpha\beta} , \qquad (8)$$

$$\Gamma_{\alpha\mu,\nu} = \Gamma_{\mu\alpha,\nu} = - \Gamma_{\mu\nu,\alpha} = \tfrac{1}{2} F_{\mu\nu,\alpha} \qquad (9)$$

$\Gamma_{\mu\nu,\sigma}$ = (the Christoffel symbols of $g_{\mu\nu}$ on M).

Thus at $p \in P$ the geodesic equations are

$$\frac{D\dot{x}_\nu}{dt} = F^{\hat{a}}_{\nu\mu} \dot{x}^\nu \dot{x}_{\hat{a}} + \tfrac{1}{2} D_\nu g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta , \qquad (10)$$

$$\frac{d\dot{x}^\alpha}{dt} + g^{\alpha\beta} \dot{x}^\mu (D_\mu g_{\beta\gamma}) \dot{x}^\gamma = g^{\alpha\beta} C_{\beta\gamma,\delta} \dot{x}^\gamma \dot{x}^\delta . \qquad (11)$$

By comparing Eqs (4) and (10) we see that the Higgsonic charge $Z_{\alpha\beta}$ is not an independent quantity. We should have

$$Z^{\alpha\beta} \approx \dot{x}^{\alpha}\dot{x}^{\beta} \quad , \tag{12}$$

and

$$q_{\hat{a}} \approx g_{\hat{a}\hat{b}}\ \dot{x}^{\hat{b}} \tag{13}$$

The difficulty in interpreting the equations (11) is that an observer in M which is blind to the extra dimensions G∖H will not distinguish between geodesics γ and γh , where h is any element of the stability group H. Both geodesics pass through the same point $p \in P$ and have the same projection on M, nevertheless the vertical components $\dot{x}^{\alpha}$ (taking values in G/H) differ by the transformation Ad(h). The difficulty does not arise for the components $\dot{x}^{\hat{a}}$ since vectors in K = Lie(K) are Ad(H)-invariant. However, the remaining Higgs charges Z_{ab} should be understood as being built out of the H-orbits $[q^{a}]$ rather than vectors $q^{a} \in L$. The least trouble for an interpretation gives the coupling $Z^{ab}D_{\nu}g_{ab}$ in (10). Indeed, owing to the Ad(H)-invariance of $g_{\alpha\beta}$ [1] we find that $q^{a}q^{b}D_{\nu}g_{ab}$ depends on the equivalence class (H-orbit) [q] of q only. Also the coupling on the right hand side of (5) which gives rise to the coloured charge nonconservation, can be described in terms of equivalence classes [q] without any difficulties. The interpretation of the remaining geodesic equations (11) for α=a in terms of H-based quantities is given in [4], where evolution equation for the nonlinear Higgs charge $[q^a]$ is discussed. An alternative approach can be based on the following observation: among the symmetric tensors $Z^{ab} = q^a q^b$, where q runs through the orbit $[q]$, there is exactly one which is Ad(H)-invariant. Thus, as far as couplings via Z^{ab} are important, one could try to eliminate the nonlinear $[q]$-s completely. However it is difficult to see how (11) could give an evolution of the composite Z^{ab} without being forced to use its (nonunique) "square root" q^a. Therefore the nonlinearity of the Higgs charges $[q]$ seems to be unavoidable.

References

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