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Particle Tracks, Events and Quantum Theory

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The law of track formation in cloud chambers is derived from the Liouville equation with a simple Lindblad's type generator that describes coupling between a quantum particle and a classical, continuous medium of two-state detectors. Piecewise deterministic random process (PDP) corresponding to the Liouville equation is derived. The process consists of pairs (*classical event, quantum jump*), interspersed with random periods of continuous (in general, non-linear) Schrödinger-type evolution. The classical events are flips of the detectors—they account for tracks. Quantum jumps are shown, in the simplest, homogeneous case, to be identical to those in the early spontaneous localization model of Ghirardi, Rimini and Weber (GRW). The methods and results of the present paper allow for an elementary derivation and numerical simulation of particle track formation and provide an additional perspective on GRW's proposal.

§ 1. Introduction

Inspired by Bell's challenging call for an exact formulation of quantum measurement theory,[1),2)] Blanchard and the present author proposed a model of quantum measurement based on completely positive (CP) semigroup coupling between a quantum system and a classical one.[3)] The main advantages of this proposal emerged only after the publication of 3). In the following series of papers[4)~7)] the method of Ref. 3) was successfully applied to several model physical situations, including Zeno's effect, Stern-Gerlach-type coupling, particle position detector and SQUID-tank system. In all those cases the coupling was shown to lead to a piecewise-deterministic random process (PDP) describing time series of experimentally observed events. Moreover, in Ref. 8) models that deal with simultaneous measurement of several non-commuting observables were described, and it was suggested that the question of determining an unknown state of the quantum system should be answered using the proposed exact definition of a measurement. However, the obvious and crucial test of any quantum measurement theory, namely, that of finding the laws governing track formation in cloud chambers and on photographic plates was, until recently, missing. The reason for this was partly of a technical character, namely, in all of these previous applications the classical system was either discrete or finite-dimensional, otherwise technical difficulties mounted. In the present paper we will show how these difficulties can be overcome owing to the discrete Poisson nature of the PDP.

Technically, the paper is concerned with a non-relativistic quantum particle

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coupled to a classical medium of two-state particle detectors. The medium is characterized by a family of "sensitivity" functions $g_a(x)$, where g_a can be thought of as a Gaussian-like function centered at a.*) The configuration space of the classical system is, in general, infinite-dimensional. In § 3 we will write down the simplest possible Liouville equation (Eqs. (3·5)~(3·7)) corresponding to the intuitive idea that presence of the particle at some point a causes flip in the detector located at that point. The functions g_a are used to describe the spatial sensitivity, and also the response time, of the detectors. The quantum Hamiltonian is allowed to depend on the actual configuration of the medium (although in most applications such a dependence can be neglected). We denote by H_Γ the Hamiltonian corresponding to detectors flipped at the points of a set Γ. The main result of the present paper is the derivation of the PDP corresponding to this coupling. For simplicity we will consider only the case where g_a and H_Γ do not depend explicitly on time. Generalization to the time-dependent case is however straightforward, and the formulation below covers also this more general case. The PDP, derived in § 4, can be summarized as follows:

Denote

$$\Lambda(x)=\int g_a(x)^2 da\,, \tag{1·1}$$

$$\tilde{H}_\Gamma=H_\Gamma-\frac{i}{2}\Lambda\,. \tag{1·2}$$

Suppose one starts, at time t_0, with all detectors in the "off" state, except those in a finite set Γ_0, and with the quantum object described by a wave function $\psi_0=\psi_{t_0}$. Then ψ evolves continuously according to the modified Schrödinger evolution:

$$\psi_t=\tilde{\psi}_t/\|\tilde{\psi}_t\|\,, \tag{1·3}$$

where $\tilde{\psi}_t$ is the solution of

$$i\frac{d}{dt}\tilde{\psi}_t=\tilde{H}_{\Gamma_0}\tilde{\psi}_t \tag{1·4}$$

with the initial condition $\tilde{\psi}_{t_0}=\psi_0$, until a jump occurs at a random time t_1, at which time the wave function is, say, ψ_{t_1}. The jump consists of a pair: (classical event, quantum jump). The classical part is a flip of the detector state at a random point of space, say at a_1. It happens at a point a, with the probability density $p(a)$ given by $p(a)=\|g_a\psi_{t_1}\|^2/\lambda(\psi_{t_1})$, where the rate function λ is given by $\lambda(\psi)=(\psi, \Lambda\psi)$. The quantum part of the jump is jump of the Hilbert space vector ψ_{t_1} to the new state $\psi_1=g_{a_1}\psi_{t_1}/\|g_{a_1}\psi_{t_1}\|$. After the jump the process starts again with a continuous time evolution as before, but now with t_0 replaced by t_1 and Γ_0 replaced by

$$\Gamma_1=\Gamma_0\Delta\{a_1\}\,,$$

where Δ denotes the set-theoretical symmetric difference. After n events that happened at the points $a_1, \cdots, a_n$, one puts

*) If there is no detector at a, we put $g_a(x)\equiv 0$. Thus our model covers also the case of a discrete, finite or infinite, number of detectors.

$$\Gamma_n = \Gamma_0 \Delta\{a_1\}\Delta \cdots \Delta\{a_n\}\,.$$

The random times of jumps are regulated by an inhomogeneous Poisson process: the probability $P(t, t+dt)$ for the first jump to occur in the time interval $(t, t+dt)$ is computed from the formula

$$P(t, t+dt) = 1 - \exp\left(-\int_t^{t+dt} \lambda(\psi_s) ds\right) \approx \lambda(\psi_t) dt\,. \tag{1·5}$$

Our model admits an interesting special case—that of a passive, homogeneous medium. If the medium is passive, i.e., if the quantum Hamiltonian does not depend on the actual state of the medium, and if it is homogeneous, then the description simplifies: the quantum process separates, the jump rate is constant and one gets "spontaneous wave-packet reductions" of Ghirardi-Rimini-Weber (cf. e.g., Ref. 9) and references therein). In general, however, the process of formation of a track has a non-constant rate, and the dependence of the rate of jumps on the state of the quantum system given by the present model is essential and experimentally verifiable.*) We believe that the proposed model of the particle track formation is the simplest one that gives intuitively expected results. It can be used for numerical simulation of particle track formation for different Hamiltonians and for different geometric configurations. It should be, in particular, interesting to analyze numerically the influence of particle detectors on sharpness of the fringe pattern in interferometry experiments.

From a philosophical point of view, it is worth noting that in the present paper, in sharp contrast to the standpoint taken by Stapp in his recent paper,[11] we deliberately avoid the concepts of an "observer". Our model aims at being as objective as the concept of probability allows for it. A philosophical summary of our results can be formulated as follows: Quantum Theory, once invented by human minds and once asked questions that are of interest for human beings, does not need "minds" or "observers" any more. What it needs is a lot of computing power and effective random number generators, rather than "observers". The fundamental question, to which we do not know answer yet, can be thus formulated as follows: can random number generators be avoided and replaced by deterministic algorithms of a simple and clear meaning?

§ 2. Events and quantum measurements

In this paragraph we will briefly describe the main ideas that influenced our way of looking at the quantum mechanical measurement problem, and that finally led to the simple cloud chamber model of this paper.

The crucial concept of our approach to quantum measurements is that of an "event". The importance of this concept, and the intrinsic incapability of quantum theory to deal with it, have been stressed by several authors. In 1958 Schrödinger

*) This is one of the important differences between our approach and other ones, where dependence of the timing of wave packet reductions on the actual state of the quantum system could not be derived—cf. e.g., Ref. 10) and references therein.

wrote:[12)]

> 'It is usually believed that the current orthodox theory actually accounts for the "nice linear traces" produced in the Wilson chamber, etc. I think this is a mistake, it does not.'

Stapp stressed the role of "events" in the, "world process" (Refs. 13) and 14), cf. also the entry "events" in the Index of Ref. 15)). Chew used Stapp's ideas on soft-photon creation-annihilation processes (cf. 16)) and proposed the term "explicate order", complementing Bohm's "implicate" quantum order, to denote the world process of "gentle" creation-annihilation events.[17)] Haag emphasized[18)] that "an event in quantum physics is discrete and irreversible" and that "we must assume that the arrow of time is encoded in the fundamental laws ⋯". In Ref. 19) he went on to suggest that "transformation of possibilities into facts must be an essential ingredient which must be included in the fundamental formulation of the theory".

In 1) and 2) Bell reprimanded the physics community for misleading use of the term "measurement" in quantum theory. He opted for banning this word from our quantum vocabulary, together with other vague terms such as "macroscopic", "microscopic", "observable" and several others. He suggested to replace the term "measurement" by that of "experiment", and also not speak of "observables" (the things that seem to call for an "observer") but to introduce instead the concept of "beables" —the things that objectively "happen-to-be (or not-to-be).[*)]

On the technical side, Machida and Namiki[20)] proposed a way of describing measurements in quantum mechanics that inspired Araki[21),22)] to formulate his continuous superselection rule model of classical measuring apparatus in quantum mechanics. In Araki's model infinite time was, however, needed for an "event" (change of the classical pointer position) to occur.

In a series of papers Sudarshan et al. investigated possibility of solving the measurement problem via a unitary, Hamiltonian coupling between a quantum and a classical system (cf. Ref. 23) and references therein).

Landsman[24)] and Ozawa[25)] gave quite general ("no-go") arguments that stressed impossibility of coupling of classical and quantum degrees of freedom via a unitary, finite-time dynamics.[**)]

On the other hand many authors were using "dynamical semigroups" —*non-unitary* dissipative time-evolutions that described an effective dynamics of quantum systems coupled to other quantum systems or to external "reservoirs" or "environment". Gorini et al.[26)] and Lindblad[27)] derived a general form of generators of norm-continuous semigroups of completely positive maps of the operator algebra of a Hilbert space.[***)] Such semigroups were widely applied to many kinds of "master equations" of statistical physics, while Ghirardi, Rimini and Weber[9)] proposed to use a particular Lindblad-type generator for describing a "spontaneous reduction proc-

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*) Calling observables "observables" can be, however, justified in the framework of an "objective theory of experiments". We plan to discuss this subject elsewhere.

**) A short no-go argument can be found in Ref. 8).

***) It was later extended by Christensen and Evans[28)] to cover the case of more general operator algebras, including the case that is most interesting for us—that of a non-trivial centre.

ess" for a single quantum particle. The GRW model incorporated "quantum jumps" that occurred in finite (Poisson distributed) times, but it did not account for the (classical) "events". Although it was clear to the experts that using dissipative semigroups instead of a unitary dynamics allowed to go around the no-go theorems, it is only in 3) that simple methods of construction of dissipative generators were found that led to measurement-like couplings of quantum and classical degrees of freedom. Later on, in Refs. 4) and 8), using the results of Davis (see Refs. 29) and 30)), a piecewise-deterministic random process (PDP) on the space of pure states of the total (classical+quantum) system was associated with the Liouville equation. While the Liouville equation describes continuous time-evolution of density matrices, that is of statistical states that concern *ensembles*, the associated piecewise-deterministic random process contains apparently more useful information: it can be used to simulate real-time behaviour of *individual* systems in measurement-like situations.

§ 3. The cloud chamber model

Our aim is to explain the "nice linear tracks" that quantum particles leave on photographs and in cloud chambers. These tracks are indeed hard to explain if one assumes that there are no particles and no events—only Schrödinger's waves. Schrödinger himself was perplexed and not quite sure which way to take.

Physically, a photographic plate or a cloud chamber is a highly complex many-particle system. Physiologically, it appears to exhibit a complex, irreversible dynamics to an external living observer. Many factors participate in the result —including the mediation of photons in the final act of perception. However, it seems to us that the detailed internal structure of local particle detectors, and also the details of the perception process, would it be human or animal, are totally irrelevant for the phenomenon itself. What is relevant, it is the response of the detectors to the quantum particle, and their back reaction on it. We put forward conjecture that it is sufficient to assume that we have to do with a system of *classical* two-state detectors that can change their state when a particle passes nearby. Although the real cloud chamber have a finite number of sensitive centers, it proves to be no more difficult to deal with a more general, continuous model—the extra bonus being that we cover this way the GRW model as well.

There is a formal detail in the model below that deserves to be mentioned: our model is more *reversible* than any real cloud chamber. Namely, we allow for a local detector to change its state *back*, when it registers the particle for the second time, and so on. This makes the model slightly easier to solve.*) The present model can be easily reformulated to cover also the case of "only-one-flip" detectors. The final PDP proves to be the same except that each detector can flip only ones.

The derivation of the model below is heuristic. Nevertheless it leads to a well-defined piecewise-deterministic random process that has a clear physical meaning. We then show that for a passive, homogeneous, medium, the effective time evolution of the quantum system itself happens to be also Markovian—it is described

*) On the other hand, it is related to the so-called, "detailed balance condition" that is often postulated in statistical physics models—for a recent discussion, cf. Ref. 31) and references therein.

by an effective CP semigroup that is identical to that postulated by Ghirardi et al.[9] This fact may suggest another application of our model: instead of considering it as an approximate model of a real, discrete and finite, cloud chamber, we may consider it as an exact model of some, perhaps yet unknown, space-structure that is participating in a universal process of wave packet reductions. The actual physical interpretation may depend on the values of parameters that enter the model. There will be, essentially, two free parameters: a coupling constant λ, of physical dimension t^{-1}, that will regulate the expected time rate of jumps, and a normalized Gaussian function whose width determines space sensitivity of the detectors. In fact, aiming at a wider applicability of our model, we will allow for non-constant rates of jumps, and for more general, not necessarily Gaussian, sensitivity functions. Clearly, presence of arbitrary functions that are external to the model, makes it to look like a phenomenological rather than as a fundamental description—unless these functions are derived from geometrical and probabilistic considerations.

We proceed to describe our model in mathematical terms. The description will be brief and will never go beyond elementary mathematical concepts. Special mathematical terms, when they occur, are used only in an informal way and can be skipped by a reader who is mainly interested in the main ideas and results. Let E denote the physical space, we take for definiteness $E=\boldsymbol{R}^n$, although it is straightforward to assume E to be a homogeneous space or an arbitrary Riemannian manifold. We consider the space E filled up with a continuous medium which can be, at each point $a\in E$, in one of its two states: "on-state", represented by $\binom{1}{0}$, or "off-state", represented by $\binom{0}{1}$. We would like to consider the set of all possible states of the medium. This is however enormously big a set, because states of the medium are, in our case, in one-to-one correspondence with its configurations, that is with subsets of E. Indeed, to each state of the medium we can uniquely associate the set of all points that are "on". Thus the set of all states of the medium is isomorphic to 2^E. Fortunately we can restrict our attention to much smaller classes of subsets of E. Let us introduce equivalence relation "$\sim$" in 2^E, with equivalence classes consisting of subsets of E that differ one from another by at most finite number of elements. Denoting by Δ the set-theoretical symmetric difference operation, we have: $\Gamma\sim\Gamma'$ if and only if $\Gamma\Delta\Gamma'$ is a finite set. It will be sufficient for us to choose some "ground state" and to take its equivalence class, that is the set of these configurations that differ from the "vacuum" in at most finite number of points. For convenience we will take for the ground state the state of "all off", represented by the empty subset $\emptyset\in 2^E$. Its equivalence class $\mathcal{S}=[\emptyset]$ consists of those states of the medium that are everywhere "off" except in a finite number of points, i.e., the class of all finite subsets of E.

Remark The fact that we can restrict ourselves to the above class $\mathcal{S}$ of sets, instead of dealing with whole of 2^E, is not evident by itself. It will be justified only á posteriori, when we will see that the "events", that will appear in the piecewise-deterministic random process which we will construct later on, consist of "flipping" a state of the medium in single (randomly chosen according to appropriate probability distribution) points of E, and that with probability one there is a finite number of events in any finite interval of time.

We can endow $\mathcal{S}$ with a topology and with a measurable structure as follows: first of all we observe that $\mathcal{S}$ is a disjoint union of subsets $\mathcal{S}_i$, $i=0, 1, \cdots$, where $\mathcal{S}_i$ consists of those states that differ from the ground state at exactly i points of E. But then, $\mathcal{S}_i$ is isomorphic to the i-th Cartesian power of E, with coinciding points extracted, and divided by the action of the permutation group in i elements. It follows in particular that $\mathcal{S}$ has a power of the continuum.

Statistical states of the classical system are probability measures on $\mathcal{S}$. They are represented by sequences $\{\mu_i\}$, where μ_i is a measure on $\mathcal{S}_i$, and $\sum_{i=0}^{\infty} \mu(\mathcal{S}_i)=1$.

Let $\mathcal{H}_q=L^2(\boldsymbol{R}^n, d^n x)$ be the Hilbert space that is used for description of the quantum system coupled to our classical medium. We denote by $\mathcal{B}(\mathcal{H}_q)$ the algebra of bounded linear operators on $\mathcal{H}_q$. Its elements are "observables" of the quantum system. Statistical states of the quantum system are normalized (by $\mathrm{Tr}(\rho)=1$) positive trace class operators on $\mathcal{H}_q$. Then statistical states of the total, classical *plus* quantum, system are described by measures ρ on $\mathcal{S}$ with values in positive, trace class, operators on $\mathcal{H}_q$, with $\sum_{i=0}^{\infty}\mathrm{Tr}(\rho(\mathcal{S}_i))=1$. A natural candidate for the algebra $\mathcal{A}_{\mathrm{tot}}$ of observables of the total system is the algebra of continuous, bounded functions on $\mathcal{S}$ with values in $\mathcal{B}(\mathcal{H}_q)$. Thus, $\mathcal{A}_{\mathrm{tot}}$ is the direct sum of algebras $\mathcal{A}_i$, where $\mathcal{A}_i$ is the algebra of continuous, bounded, $\mathcal{B}(\mathcal{H}_q)$-valued functions on $\mathcal{S}_i$. As our main aim is to derive the PDP rather than to prove the existence of CP semigroup—we will apply, from now on, a heuristic notation. Thus, a state of the total system will be represented by a family $\{\rho_\Gamma\}_{\Gamma\in\mathcal{S}}$, with '$\sum_\Gamma$'$\mathrm{Tr}(\rho_\Gamma)=1$.

To have some definite example in mind, in what follows we will take for the quantum system a particle of mass m moving in $E=\boldsymbol{R}^n$ according to the dynamics described by the quantum Hamiltonian

$$H_\Gamma=-\frac{\hbar^2}{2m}\left(\nabla_x-\frac{e}{\hbar c}\boldsymbol{A}_\Gamma\right)^2+V_\Gamma(x)\,. \tag{3·1}$$

We thus allow quantum Hamiltonian to depend on the actual state of the medium.

Remark We could allow H to depend explicitly on time—then the semigroup property would be lost, but PDP would be described in the same way as in the present model. Generalization to the case of quantum particle moving on a manifold and acted upon by gravitational and electromagnetic forces is straightforward. A more general treatment, including Bose or Fermi multiparticle case, will appear elsewhere.[32] The idea will not change also in such a case.

We proceed now to describe the coupling that corresponds to the following intuitive picture: *the medium consists of detectors that can change their state if the particle approaches them sufficiently close for a sufficient time.* Space and time sensitivities of the detectors are described by real, non-negative functions $g_a(x)$, where the variable a describes the position of the detector. We can think of g_a as a hat-like function with its center at $x=a$. We introduce then the non-negative function $\Lambda(x)$ defined by

$$\Lambda(x)=\int_E g_a(x)^2 da \tag{3·2}$$

for all $x\in E$. Here da denotes the Lebesgue measure, but if we want to describe a discrete, rather than a continuous, case, then the integral above should be replaced by a sum. By the abuse of notation we will denote by the letter Λ the operator of multiplication by the function $\Lambda(x)$, acting on the Hilbert space $L^2(\boldsymbol{R}^n, d^n x)$.

Each density matrix ρ of the total system can be, formally, written as

$$\rho = \sum_{\Gamma \in \mathcal{S}} \rho_\Gamma \otimes \epsilon_\Gamma \,, \tag{3·3}$$

where, for $\Gamma \in \mathcal{S}$,

$$\epsilon_\Gamma = \prod \otimes_{a \in E} \begin{pmatrix} \chi_\Gamma(a) & 0 \\ 0 & 1-\chi_\Gamma(a) \end{pmatrix}, \tag{3·4}$$

and where χ_Γ stands for the characteristic function of the set Γ, i.e., $\chi_\Gamma(a)=1$ for $a \in \Gamma$, otherwise $\chi_\Gamma(a)=0$.

Remark The last statement requires some care. It is also not quite trivial. For a finite number of detectors it is not too difficult to see. We are using the above notation introduced by Neumann in his theory of continuous tensor products. To give to the above expressions a precise mathematical meaning, we would have to invoke a part of this theory. (For a more modern account, cf. Ref. 33) and references therein.) That tool is however not necessary for the present, heuristic, purpose. More complete mathematical treatment will be given elsewhere.

To define the coupling between the particle and the medium, we will apply the ideas introduced in Refs. 3) and 4). Namely, we will write the Liouville time evolution equation for the statistical state of the total system as

$$\dot{\rho} = -i[H, \rho] + \mathcal{L}(\rho) \,, \tag{3·5}$$

where $\mathcal{L}$ is a Lindblad-type generator that provides dissipative coupling.

Remark In the present model we will neglect a possible free dynamics of the medium.

For $\mathcal{L}$ we take the simplest possible coupling:

$$\mathcal{L}(\rho) = \int da \left(V_a \rho V_a - \frac{1}{2}\{V_a^2, \rho\}\right), \tag{3·6}$$

where

$$V_a = g_a \otimes \tau_a \,, \tag{3·7}$$

g_a being the multiplication operator by the function $g_a(x)$, and τ_a denoting the "flip" of the detector at the point a:

$$\tau_a = \prod \otimes_b u_b \,, \tag{3·8}$$

where

$$u_b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3·9}$$

for $b \neq a$, while

$$u_a = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3·10}$$

Because $\tau_a^2 = Id$, our evolution equation reads now:

$$\dot{\rho} = -i[H, \rho] + \int da\, V_a \rho V_a - \frac{1}{2}\{\Lambda, \rho\}, \tag{3·11}$$

where Λ in the anticommutator is understood as a multiplication operator by the function $\Lambda(x)$.

Let us denote by $a(\Gamma)$ the set representing the state Γ with the flipped a:

$$a(\Gamma) = \Gamma \Delta \{a\}. \tag{3·12}$$

Then, using change of summation variable $\Gamma \to \Gamma' = a(\Gamma)$, and also using the fact that $a(a(\Gamma)) = \Gamma$ —i.e., that the second flip cancels the first, we obtain

$$V_a \rho V_a = \sum_{\Gamma \in S} g_a \rho_\Gamma g_a \otimes \epsilon_{a(\Gamma)} = \sum_{\Gamma \in S} g_a \rho_{a(\Gamma)} g_a \otimes \epsilon_\Gamma \tag{3·13}$$

so that we can write:

$$\dot{\rho}_\Gamma = -i[H_\Gamma, \rho_\Gamma] + \int da\, g_a \rho_{a(\Gamma)} g_a - \frac{1}{2}\{\Lambda, \rho_\Gamma\}. \tag{3·14}$$

The equation (3·14) is fundamental. It describes time evolution of the family $\{\rho_\Gamma\}$, where Γ runs over all finite subsets of E. All the relevant statistical information about the quantum particle and the classical medium can be derived from this equation. In the next paragraph we will derive the piecewise deterministic random process that is compatible with Eq. (3·14) and that concerns histories of individual coupled systems. Before however going to this, let us see that in the passive, homogeneous, case we can obtain effective time evolution for the quantum particle alone. If the medium is passive, then the Hamiltonian does not depend on the actual state of the medium: $H_\Gamma \equiv H$. If the medium is homogeneous then, for symmetry reasons, Λ must be a constant: $\Lambda(x) \equiv \lambda$. For instance this happens if we take for g_a the Gaussian functions:

$$g_a(x) = \lambda^{1/2} \left(\frac{\alpha}{\pi}\right)^{n/2} \exp(-\alpha(x-a)^2). \tag{3·15}$$

The effective state of the quantum particle is determined by tracing over the classical configurations:

$$\hat{\rho} = \sum_{\Gamma \in S} \rho_\Gamma. \tag{3·16}$$

To sum up Eq. (3·14) over Γ we note that, for each $a \in E$, $a: \Gamma \mapsto a(\Gamma)$ is a one-to-one map of S onto itself, this owing to the fact that 2^E is a group under the symmetric-difference operation, and that S is a subgroup.

Thus we have $\sum_{\Gamma \in S} \rho_{a(\Gamma)} = \sum_{\Gamma \in S} \rho_\Gamma = \hat{\rho}$. It follows that the time derivative of $\hat{\rho}$

depends, for our particular choice of the coupling, only on $\hat{\rho}$ and not on the full hierarchy of the ρ_Γ's; we have

$$\dot{\hat{\rho}}=-i[H, \hat{\rho}]+\int da g_a \hat{\rho} g_a - \lambda \hat{\rho} , \qquad (3\cdot 17)$$

which is exactly of the type discussed by Ghirardi, Rimini and Weber (cf. Ref. 9)).

§ 4. The Piecewise deterministic process

4.1. *Definition of PDP and its infinitesimal generator*

In his monographs[29),30)] dealing with stochastic control and optimization Davis, having in mind mainly queuing and insurance models, described a special class of piecewise deterministic processes that was later found to fit perfectly the needs of quantum measurement theory. Even if for the present model we will have to extend slightly the original Davis' framework, and to work with jumps between continuously parameterized states and not between discrete manifolds, we will describe briefly the discrete case and we leave the problem of a rigorous formulation of its evident extension to continuous families aside.

Let ι be an index running over a finite or countable set J. Consider functions $f(\xi, \iota)$, where for each ι the variable ξ is continuous and runs through some set M.[*)] Suppose we have a semigroup of transformations α_t acting on the space of such functions with the infinitesimal generator $\mathcal{D}$ which is an integro-differential operator of the following form:

$$(\mathcal{D}f)(\xi, \iota)=(Z_\iota f)(\xi, \iota)+\lambda(\xi, \iota)\Sigma_{\iota'}\int_M Q(\xi, \iota; d\xi', \iota')(f(\xi', \iota')-f(\xi, \iota)) , \qquad (4\cdot 1)$$

where Z_ι are vector fields that generate one-parameter flows ϕ_ι on M, $\lambda(\xi, \iota)$ are non-negative functions, while $Q(\xi, \iota; d\xi', \iota')$ are (non-negative) transition measures —thus satisfying

$$\sum_{\iota'}\int_M Q(\xi, \iota; d\xi', \iota')=1 , \qquad (4\cdot 2)$$

and also

$$\int_{\{\xi\}} Q(\xi, \iota; d\xi', \iota)=0 \qquad (4\cdot 3)$$

for all ι and $\xi\in M$. We notice that by the very definition we have $Z_\iota(\xi) = d\phi_\iota(\xi, t)/dt|_{t=0}$. Then, as it is shown in Refs. 29) and 30), one can associate with this generator $\mathcal{D}$ a piecewise deterministic stationary Markov process that is described as follows.

Suppose the process starts at some point (ξ_0, ι_0). Then ξ evolves continuously along the vector field Z_ι, $\xi_t=\phi_\iota(\xi_0, t)$, while ι_0 remains constant until a jump occurs at a certain random time t_1. The time of this jump is governed by a (inhomogeneous)

*) We will need the case where also ι will be continuous running over E, while M will coincide with the unit ball in the Hilbert space $L^2(E)$ (modulo the phase).

Poisson process with rate function $\lambda(t)=\lambda(\xi_t, \iota_0)$. When jump occurs at $t=t_1$, then (ξ_{t_1}, ι_0) jumps to (ξ', ι) with probability density $Q(\xi_{t_1}, \iota_0; d\xi', \iota)$ and the process starts again.

Remark Notice that the probability that the jump will occur between t and $t+dt$, provided it did not occur yet, is equal to $1-\exp(-\int_t^{t+dt}\lambda(s)ds)\approx\lambda(t)dt$. This justifies calling λ the rate function.

Association of the random process with the semi-group α_t is canonical and can be described as follows: first one goes from α_t that acts on functions $f(\xi, \iota)$ to its dual α^t acting on measures. Then, choosing the Dirac measure δ_{ξ_0,ι_0} concentrated at (ξ_0, ι_0) as the initial point μ_0, we apply to it α^t to get $\mu_t=\alpha^t(\mu_0)$. The resulting measure μ_t is then characterized by the fact that $d\mu^t(\xi, \iota)$ is equal to the probability that the process starting at $t=0$ from (ξ_0, ι_0) will end, at time t, at the point (ξ, ι).

A detailed and precise description of the above correspondence should include specification of the involved measure structures and domains of definition. We refer the reader to Refs. 29) and 30) for mathematical details. Here we only notice the following important relation: let $K(t; \xi, \iota; d\xi', \iota')$ be the transition function for the process. Then the semigroup α_t is given by the formula

$$(\alpha_t f)(\xi, \iota)=\sum_{\iota'}\int K(t; \xi, \iota; d\xi', \iota')f(\xi', \iota'). \tag{4·4}$$

4.2. *Derivation of the PDP for the cloud chamber model*

We will now describe the most important fact about our cloud chamber model: we will show that Eq. (3·14) describing the time evolution of statistical states of the total system can be interpreted in terms of a piecewise deterministic Markov process. That process has then a transparent description in terms of pairs of (*classical event, quantum jump*) that are interspersed (in a random way, according to an inhomogeneous Poisson point process law with rate λ) with the periods of continuous, Schrödinger's type, time evolution. In our derivation we will consider the case where H_Γ and g_a do not depend explicitly on time. In this case the process defined in the Introduction is a stationary Markov process, and we are dealing with semigroups. It is however straightforward to generalize our reasoning to non-stationary case. The derivation and the formulas are in this case much the same, except that semigroups should be replaced by two-parameter families of maps, and that in several evaluations below $t=0$ should be replaced by an arbitrary t.

If we want to interpret Eq. (3·14) in terms of a PDP on pure states, then the first thing we have to do, is to rewrite Eq. (3·14) as an equation for observables rather than states. After doing so we will interpret observables as functions on pure states.

Given a state $\rho=\{\rho_\Gamma: \Gamma\in S\}$ and an observable $A=\{A_\Gamma: \Gamma\in S\}$, the expectation value of A in ρ is given by $\langle\rho, A\rangle=\sum_\Gamma \mathrm{Tr}(\rho_\Gamma A_\Gamma)$. Time evolution of observables is then defined as dual to the time evolution of states, so that we have $\langle\rho, \dot{A}\rangle=\langle\dot{\rho}, A\rangle$. By substituting the equation (3·14) for $\dot{\rho}$, we easily find that, in our case, observables evolve according to the law that is almost identical to that for states, except that there is change of sign in front of the commutator:

$$\dot{A}_\Gamma = i[H_\Gamma, A_\Gamma] + \int da g_a A_{a(\Gamma)} g_a - \frac{1}{2}\{\Lambda, A_\Gamma\}. \tag{4·5}$$

Each observable A (of the total system) can be interpreted as a function f_A on pure states (of the total system):

$$f_A(\psi, \Gamma) \equiv (\psi, A_\Gamma \psi), \quad \psi \in \mathcal{H}_q, \Gamma \in S. \tag{4·6}$$

We can now sandwich Eq. (4·5) between two ψ vectors to see if we can interpret this equation in terms of time evolution of functions on pure states. We get

$$\begin{aligned} \dot{f}_A(\psi, \Gamma) &\equiv f_{\dot{A}}(\psi, \Gamma) \\ &= (\psi, i[H_\Gamma, A_\Gamma]\psi) + (\psi, \textstyle\int da g_a A_{a(\Gamma)} g_a \psi) - \frac{1}{2}(\psi, \{\Lambda, A_\Gamma\}\psi). \end{aligned} \tag{4·7}$$

The first term on the rhs of Eq. (4·7) can be written also as $(Z_H f_A)(\psi, \Gamma)$, where Z_H is the vector field of the Hamiltonian evolution of pure states

$$(Z_H f)(\psi, \Gamma) \doteq \frac{d}{dt} f(e^{-iH_\Gamma t}\psi, \Gamma)\Big|_{t=0}. \tag{4·8}$$

The second term can be rewritten as

$$\begin{aligned} (\psi, \textstyle\int da\, g_a A_{a(\Gamma)} g_a\, \psi) &= \int da (g_a \psi, A_{a(\Gamma)} g_a \psi) \\ &= (\psi, \Lambda\psi) \int da \frac{\|g_a\psi\|^2}{(\psi, \Lambda\psi)} f_A\left(\frac{g_a\psi}{\|g_a\psi\|}, a(\Gamma)\right). \end{aligned} \tag{4·9}$$

Finally, the third term of Eq. (4·7), rewritten in terms of the functions f_A, gives rise to two terms:

$$\begin{aligned} -\frac{1}{2}(\psi, \{\Lambda, A_\Gamma\}\psi) &= \frac{d}{dt}\left(\left\|\exp\left(-\frac{\Lambda}{2}t\right)\psi\right\|^2 f_A\left(\frac{\exp\left(-\frac{\Lambda}{2}t\right)\psi}{\left\|\exp\left(-\frac{\Lambda}{2}t\right)\psi\right\|}, \Gamma\right)\right)\Bigg|_{t=0} \\ &= -(\psi, \Lambda\psi) + \frac{d}{dt}\left(f_A\left(\frac{\exp\left(-\frac{\Lambda}{2}t\right)\psi}{\left\|\exp\left(-\frac{\Lambda}{2}t\right)\psi\right\|}, \Gamma\right)\right)\Bigg|_{t=0}. \end{aligned} \tag{4·10}$$

Let us introduce the second vector field Z_D corresponding to the non-linear evolution:

$$(Z_D f)(\psi, \Gamma) \doteq \frac{d}{dt} f\left(\frac{\exp\left(-\frac{\Lambda}{2}t\right)\psi}{\left\|\exp\left(-\frac{\Lambda}{2}t\right)\psi\right\|}, \Gamma\right)\Bigg|_{t=0}. \tag{4·11}$$

We now see that we can write the evolution equation for the functions f_A in the form required by Eq. (4·1) provided we introduce
the rate function:

$$\lambda(\psi) = (\psi, \Lambda\psi), \tag{4·12}$$

the vector field:

$$Z=Z_H+Z_D\,, \tag{4·13}$$

and the transition measure $Q(\psi, \Gamma; \psi', \Gamma')d\psi' d\Gamma'$ that vanishes except for

$$Q(\psi, \Gamma; \psi', a(\Gamma))=\frac{\|g_a\psi\|^2}{\lambda(\psi)}\delta\left(\psi'-\frac{g_a\psi}{\|g_a\psi\|}\right)d\psi'\,, \tag{4·14}$$

where $\delta(\psi'-\psi)d\psi'$ is a symbolic expression for the Dirac measure concentrated at ψ.

It is easy to see that the vector field $Z=Z_H+Z_D$ generates the flow $\Phi_{t,t'}$ given by

$$\Phi_{t,t_0}f(\psi_0)=f(\psi_t)\,, \tag{4·15}$$

where ψ_t is given by the formulas (1.3) and (1.4).

We now describe the piecewise deterministic process on pure states of the total system that is associated with these data. Starting with the quantum system described by an initial wave packet $\psi\in L^2(E)$, and with the initial "all off" state of the medium, ψ develops according to Eq. (4·15) until a jump occurs at random time t_1, at which time the wave packet is ψ_{t_1}. The time t_1 of the jump is governed by the inhomogeneous Poisson process that is characterized by the probability $P(t, t+dt)$ for the jump to occur in the time interval $(t, t+dt)$, provided it did not occur yet. It is given by the formula

$$P(t, t+dt)=1-\exp\left(-\int_t^{t+dt}\lambda(\psi_s)\right)ds\approx\lambda(\psi_t)dt\,. \tag{4·16}$$

The jump consists of a pair (*classical event, quantum jump*). The classical event is a flip of the detector at a random point $a\in E$. It happens at a with probability density

$$p(a)=\frac{\|g_a\psi_{t_1}\|^2}{\lambda(\psi_{t_1})}\,. \tag{4·17}$$

When the classical detector flips at some point $a=a_1$, then the quantum states jumps from its actual state ψ_{t_1} to the new state ψ_1 given by

$$\psi_1=\frac{g_{a_1}\psi_{t_1}}{\|g_{a_1}\psi_{t_1}\|} \tag{4·18}$$

and the process starts again.

It is worth noting that, for simple Gaussian packets, and for a free evolution, the most probable place for a flip to occur is at the maximum of the actual wave-function. That explains linear tracks. For more complicated geometries and dynamics —numerical computation is necessary, at least until simple general laws are found that are based on PDP.

4.3. *Additional comments and discussion*

The derivation given above, although containing all the essential steps, is still too sketchy and needs some additional comments. First of all we remark that the argument above is intended to show that the jump-event process that we have described *implies* the Liouville equation. We did not attempt to prove here that the

Liouville equation implies the process. For general Liouville equations one expects that there will be several processes that lead to the same equation. For instance, in Ref. 34) examples are discussed where the same Liouville equation can be associated either with diffussion or with a jump process—the two cases corresponding to different experimental detection schemes. On the other hand what we have in our case is a very particular type of Liouville equation that describes pure measurement-type coupling (with no extra noise) between a quantum and a classical system. For such a Liouville equation we conjecture *uniqueness*. The arguments for uniqueness of the stochastic process for a class of Liouville equations that describe measurement-like couplings are given in Ref. 35). Using the method of Ref. 35) it is possible to prove that the PDP described in the Introduction follows uniquely from the Liouville equation (3·14)—at least for infinitesimal time steps. We do not have yet a completely rigorous proof for finite time steps.

Let us now discuss some mathematical details that may help the reader to fill out some missing pieces in our short argument of the previous subparagraph. We claim that the process described in the Introduction *implies* the Liouville equation (3·14). In order to better understand this claim let us explain the precise meaning of the term *implies*, which is not quite trivial in our context. Let K be the transition function (or, more precisely, measure) of the process. Thus $K\ (s;\ \xi, \Gamma;\ d\xi', \Gamma')$ is the probability that the process starting at (ξ, Γ) at time t, will end at $(d\xi', \Gamma')$ at time $t+s$. The process defines time evolution of measures on pure states. If $\mu_\Gamma(d\xi)$ is a measure on pure states (of the total system) at time t, then it evolves into measure $(\alpha^s\mu)_{\Gamma'}(d\xi')=\sum_\Gamma\int\mu_\Gamma(d\xi)K(s;\ \xi, \Gamma;\ \xi', d\Gamma')$ at time $t+s$. Now, each measure $\mu_\Gamma(d\xi)$ defines a density matrix $\{\rho_\Gamma\}$ of the total system:

$$\rho_\Gamma=\int P_\xi\mu_\Gamma(d\xi)\,, \tag{4·19}$$

where P_ξ are projection operators onto pure states ξ. However, the association is many-to-one, as the same quantum density matrix can be decomposed in infinitely many ways into pure states. Therefore a general stochastic process with values in pure states will not induce transformation of density matrices. For this we must have the property that if μ and $\tilde{\mu}$ define the same density matrix, then $\alpha_s\mu$ and $\alpha_s\tilde{\mu}$ also define the same density matrix. By the duality between the algebra of bounded operators and the space of trace class operators the last condition is equivalent to the following one:

Consistency Condition: For each Hermitian $A\in\mathcal{A}_{\text{tot}}$ and each $s\geq 0$ there exist $A'\in\mathcal{A}_{\text{tot}}$ such that $\alpha_s f_A=f_{A'}$. If the condition holds, then A' is uniquely defined, and by setting $\alpha^s(A)\equiv A'$ we obtain: $\langle\alpha^t(A)\rangle_\rho=\langle A\rangle_{\alpha_t(\rho)}$.

It is sufficient if the above condition is satisfied infinitezimally, i.e., if for each A there exists $\mathcal{L}(A)$ such that

$$\frac{d}{ds}\alpha_s f_A|_{s=0}=f_{\mathcal{L}(A)}\,. \tag{4·20}$$

That is exactly what we have done in § 4.2: we verified that $\mathcal{D}f_A=f_{\mathcal{L}(A)}$, where $\mathcal{L}(A)$

is the r.h.s. of Eq. (4·5). In this way we have proved two things: first, the process taking values in pure states of the total system, described in the Introduction, satisfies the compatibility condition and thus defines time evolution of density matrices, and second, that time evolution of density matrices defined by the process coincides with time evolution determined by the Liouville equation (3·14).

§ 5. Summary and conclusions

We have seen that a simple coupling between quantum particle and classical continuous medium of two-state detectors leads to a piecewise deterministic random process that accounts for track formation in cloud chambers and photographic plates. For a passive, homogeneous medium the process is essentially identical to the spontaneous localization GRW model of Ref. 9). In particular all the theoretical and numerical analysis that has been done for GRW models applies also in this case.

As mentioned in the Introduction, to simulate track formations only random number generators and computing power is necessary. Our model does not involve observers and minds. This does not mean that we do not appreciate the importance of the mind-body problem. In our opinion understanding the problems of minds needs also quantum theory, and perhaps even more—that is still beyond the horizon of the present-day physics. But our model indicates that quantum theory does not need human minds. Quantum theory should be formulated in a way that involves neither observers nor minds—at least not more than any other branch of physics. Our model can be considered as a step in this direction. It can rightly be criticized as being too phenomenological to satisfy us wholly. But, provided it correctly accounts for experimental results, it can give a valuable new insight into the quantum duality of potential and actual, of waves and particles, and of determined and random.

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