EVENT-ENHANCED FORMALISM OF QUANTUM THEORY OR COLUMBUS SOLUTION TO THE QUANTUM MEASUREMENT PROBLEM

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1 INTRODUCTION

Quantum Mechanics has proved to be tremendously powerful, practical, and successful in the description of the micro-world of elementary particles, atoms and molecules. There seems to be no limit to the versality of the Schrödinger equation and to the power of Quantum Theory as an incredibly accurate computational tool for the physicist, chemist, and biologist. The progress made in the last 70 years has really been a matter of sharpening the quantum mechanical mathematical formalism rather than of our understanding of it. As Quantum Mechanics amassed success after success only a few physicists remained fascinated by the fundamental problems that remained unsolved. The proposed solutions to the quantum measurement problem by e.g. von Neumann and Wigner – are no solution at all. They merely shift the focus from one unsolved problem to another. On the other hand the predictions for the outcomes of measurements performed on statistical ensembles of physical systems are excellent. What is however completely missing in the standard interpretation is an explanation of experimental facts i.e. a description of the actual individual time series of events of the experiment. That an enhancement of Quantum Theory allowing the description of single systems is necessary is nowadays clear. Indeed advances in technology make fundamental experiments on quantum

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systems possible. These experiments give us series of events for which there are definitely no place in the original, standard version of quantum mechanics, since each event is classical, discrete and irreversible. In recent papers [1 8] we provided a definite meaning to the concepts of experiment and event in the framework of mathematically consistent models describing the information transfer between classical event-space and quantum systems. We emphasize that for us the adjective 'classical' has to be understood in the following sense: to each particular experimental situation corresponds a class of classical events revealing us the Heisenberg transition from the possible to the actual and these events obey the rules of classical logic of Aristotle and Boole. The World of the Potential is governed by quantum logic and has to account for the World of Actual, whose logic is classical. We accept both and we try to see what we gain this way. It appears that working with so enhanced formalism of quantum theory we gain a lot. [1] We proposed mathematical and physical rules to describe

- the two kinds of evolution of quantum systems namely continuous and stochastic
- the flow of information from quantum systems to the classical event-space
- the control of quantum states and processes by classical parameters.

In our event-enhanced formalism the quantum system Σ_q is coupled to a classical space Σ_c – where events do happen – and a measurement is nothing else but a particular coupling between Σ_q and Σ_c; in a simplest case – via a completely positive semigroup $\alpha_t = e^{tL}$ in such a way that information can be transferred from Σ_q to Σ_c. We consider the total system $\Sigma_{tot} = \Sigma_q \times \Sigma_c$ and the behaviour associated to the total algebra of observables

$$\mathcal{A}_{tot} = \mathcal{A}_q \otimes \mathcal{A}_c = \mathcal{L}(\mathcal{H}_q) \otimes C(X_c)$$

where X_c is the classical event-space and $\mathcal{H}_q$ the Hilbert space associated to the quantum system. The "classical" pure state corresponds to a point in X_c and the coordinates of this point corresponds exactly to the properties of Σ_c. There is no such correspondence in $\mathcal{H}_q$. The quantum state is a unique sort of entity. Time evolution of ensembles of coupled system, prepared by the same algorithm, is described by a Liouville equation in the Hilbert space $\mathcal{H}_t$ of the total system with $\mathcal{H}_t = \mathcal{H}_q \otimes L^2(X_c)$. Individual quantum systems and the classical degrees of freedom are described by pairs of pure states: a pure state of the classical system Σ_c and a pure state of the quantum system Σ_q. Time evolution of this pair is derived from the Liouville equation. It is a piecewise deterministic random Markov process. At random times distributed according to a specific inhomogeneous Poisson process jumps occur. There are jumps of the quantum state vectors and also at the same time jumps of the states of Σ_c. These second jumps we can "see" (to measure a quantity we must "look at it") and these classical events can be recorded if necessary. Sometimes these can be jumps of a "pointer" discrete positions, sometimes jumps in the pointer's velocity. Knowing this PD-process one can answer many (perhaps even all) kinds of questions about time correlations of the events as well as simulate numerically the possible histories of individual systems. What we achieved in this

[1] Referring in the title to Christopher Columbus seems to us appropriate for two reasons. First he is known mainly as the discoverer of the New World despite the fact that he was certainly not the first European to land on the coast of the Americas. But his achievement is distinguished from the earlier adventures by its consequences. Second, according to legend, in addition to being a famous navigator, Columbus also knew a thing or two about the phenomenological solution to an otherwise unsolvable problem – namely that of balancing an egg on its end on a flat table. His solution, however, albeit simple and effective, was hardly acceptable to anybody else.

way is the maximum of what can be achieved without introducing hidden variables, which is more than original orthodox interpretation gives. That is why we call our approach the "Event Enhanced Quantum Theory". Within this framework there need not be cat paradoxes anymore – cats are allowed to behave as cats; we cannot predict individual events as they are random, but we can simulate the observations of individual systems. Our formalism is therefore closely linked to practical matters and is of relevance to today's experimental Quantum Physics.

In Section 2 we will briefly describe the mathematical and physical ingredients of our class of phenomenological models. The range of applications is rather wide as will be shown in Section 3 and 4 with a discussion of Bohm's version of the EPR experiment and the discussion of a general cloud chamber model. In a particular case this model explains the extension of Quantum Theory formulated by Ghirardi, Rimini and Weber in 1986. The GRW theory is based on a process of amplification called spontaneous localization [9]. Born's interpretation of the wave function can be derived and has not to be postulated. EPR connection between a pair of distant quantum entities that have previously interacted in some way seems to involve a strange communication called by Einstein a "spooky action at a distance". We discuss this problem briefly in Section 3. Section 5 deals with some other applications and concluding remarks.

2 COUPLING QUANTUM SYSTEM TO THE CLASSICAL EVENT–SPACE

For a long time the theory of measurements in Quantum Mechanics, elaborated by Bohr, Heisenberg and von Neumann in the 1930-s has been considered as an esoteric subject of little relevance for real physics. But in the 1980-s the technology has made possible to transform "Gedankenexperimente" of the 1930-s into real experiments. This progress implies that the measurement process in Quantum Theory is now a central tool for physicists testing experimentally by high-sensitivity measuring devices the deeper aspects of Quantum Theory. Quantum mechanical measurement brings together a macroscopic and a quantum system.

Let us briefly describe the mathematical framework we will use. A good deal more can be said and we refer the reader to [1 6]. Our aim is to describe a simple model of a non-trivial transfer of information between a quantum system Σ_q coupled to a classical event–space Σ_c. To concentrate on main ideas rather than on technical details let us describe first a simple situation, namely that of a coupling corresponding to a measurement of a discrete quantum observable. In this case the classical event–space can be finite. As we shall see later in the applications it is possible to handle continuous and infinite dimensional generalization of this framework. To the quantum system there corresponds a Hilbert space $\mathcal{H}_q$. In $\mathcal{H}_q$ we consider a family of orthonormal projectors $e_i = e_i^* = e_i^2,\ (i = 1, ..., n),\ \sum_{i=1}^n e_i = 1$, associated to an observable $A = \sum_{i=1}^n \lambda_i e_i$ of the quantum mechanical system. The space of classical events is supposed to have m distinct pure states, and it is convenient to take $m \geq n$, otherwise some information about the quantum system can be lost. The algebra $\mathcal{A}_c$ of classical observables is in this case nothing else than $\mathcal{A}_c = \mathbf{C}^m$. The set of classical statistical states coincides with the space of probability measures on X_c. Using the notation $X_c = \{s_0, ..., s_{m-1}\}$, a classical state is therefore an m-tuple $p = (p_0, ..., p_{m-1}), p_\alpha \geq 0,\ \sum_{\alpha=0}^{m-1} p_\alpha = 1$. The state s_0 plays in some cases a distinguished role and can be viewed as the neutral initial state of a pointer. The algebra of observables of the total system $\mathcal{A}_{tot}$ is given by $\mathcal{A}_{tot} = \mathcal{A}_c \otimes L(\mathcal{H}_q) = \mathbf{C}^m \otimes L(\mathcal{H}_q) = \bigoplus_{\alpha=0}^{m-1} L(\mathcal{H}_q)$, and it is convenient to realize $\mathcal{A}_{tot}$ as an algebra of operators on an auxiliary Hilbert space $\mathcal{H}_{tot} = \mathcal{H}_q \otimes \mathbf{C}^m = \bigoplus_{\alpha=0}^{m-1} \mathcal{H}_q$. $\mathcal{A}_{tot}$ is then isomorphic to the algebra of block diagonal $m \times m$ matrices $A = \mathrm{diag}(a_0, a_1, ..., a_{m-1})$ with $a_\alpha \in L(\mathcal{H}_q)$. States on $\mathcal{A}_{tot}$ are represented by block diagonal matrices $\rho = \mathrm{diag}(\rho_0, \rho_1, ..., \rho_{m-1})$, where the ρ_α are positive trace class operators in $L(\mathcal{H}_q)$ satisfying moreover Σ_α Tr $(\rho_\alpha) = 1$. By taking partial traces each state ρ projects onto an effective quantum state $\pi_q(\rho)$ and an effective classical state $\pi_c(\rho)$ given respectively by $\hat{\rho} \equiv \pi_q(\rho) = \sum_\alpha \rho_\alpha$, $\pi_c(\rho) = (\mathrm{Tr}\ \rho_0, \mathrm{Tr}\ \rho_1, ..., \mathrm{Tr}\ \rho_{m-1})$. Let us consider dynamics. A nontrivial coupling between both systems is impossible without a dissipative term. The time evolution of the total system is given (in the simplest case that we consider) by a semigroup $\alpha^t = e^{tL}$ of completely positive maps of $\mathcal{A}_{tot}$ preserving hermiticity, identity and positivity - with L of the Lindblad form

$$L(A) = i[H, A] + \sum_{i=1}^{n} (V_i^* A V_i - \frac{1}{2}\{V_i^* V_i, A\}). \tag{1}$$

There is a simple method of constructing appropriate couplings. In order to couple Σ_q to Σ_c in such a way that the coupling will correspond to measurement of the given quantum observable $A = \sum_{i=1}^{n} \lambda_i e_i$, the V_i are chosen as tensor products $V_i = \sqrt{\lambda} e_i \otimes \phi_i$, where ϕ_i act as transformations on classical (pure) states. Denoting $\rho(t) = \alpha_t(\rho(0))$, the time evolution of the states is given by the dual Liouville equation

$$\dot{\rho}(t) = -i[H, \rho(t)] + \sum_{i=1}^{n} (V_i \rho(t) V_i^* - \frac{1}{2}\{V_i^* V_i, \rho(t)\}), \tag{2}$$

where in general H and the V_i may depend explicitly on time (in fact, H can also carry an index: $H \to H_\alpha$).

In [1] we propose a simple, purely dissipative Liouville operator (i.e. we put $H = 0$) that describes an interaction of $\sum_q$ and $\sum_c$, for which $m = n + 1$ and $V_i = e_i \otimes \phi_i$, where ϕ_i is the flip transformation of X_c transposing the neutral state s_0 with s_i. We show that the Liouville equation can be solved explicitly for any initial state $\rho(0)$ of the total system. The quantum probabilities are after switching on of the interaction, mirrored by the state of the classical system. Moreover we show that the effective quantum state $\hat{\rho}(t) = \pi_q(\rho(t))$ of the quantum subsystem tends for $t \to +\infty$ to a limit which coincides with the standard von Neumann-Lüders quantum measurement postulate. The model can be easily generalized (cf. Refs. [4–8]) to include measurements of fuzzy or noncommuting observables (in fact, in the cloud chamber model, discussed in Section 4, we are measuring fuzzy position of a quantum particle).

The total system $\Sigma_t = \Sigma_q \otimes \Sigma_c$ is open. Thus one can try to understand its dynamical behaviour as an effective evolution of a subsystem of unitarily evolving larger quantum systems. Although mathematically possible and studied by many authors (for a recent discussion related to quantum measurements cf. Ref. [10]) – such an enlargement is non unique and our understanding of the problem of "minimal" extensions in the case of mixed, classical+quantum, systems is still rather poor. Moreover, it is not clear at all what practical gain can be achieved this way. Therefore we prefer to extend the prevailing paradigm and learn as much as possible how to deal directly with open systems.

A complete general theory of dissipative couplings of quantum systems to classical ones does not yet exist. The best we can do is to study a lot of examples. In the following sections two characteristic situations will be presented. See Refs. [2–6] for more examples. For every example we have considered, a piecewise deterministic random process has been constructed that takes place on the state of pure states of the total system and which reproduces the Liouville equation of the total system by averaging. A theory of piecewise deterministic processes (PDP) is described in a recent book by M.H. Davis [11]. Examples of processes of this type but without a non-trivial evolution on the classical space were also discussed in a physical context [12–15]. Piecewise deterministic Markov processes enjoy deterministic dynamics punctuated by random jumps generated by a Markov stochastic structure. The model introduced by Davis contains virtually all nondiffusion models of applied probability. A PDP is determined by its local characteristics:

i) A vector field X which determines a flow Φ on the state space S

$$\frac{d}{dt}f(\Phi(t,x)) = Xf(\Phi(t,x)), \qquad \Phi(0,x) = x,$$

ii) A jump rate function λ,

iii) A transition probability matrix $\mathcal{Q}$.

From these characteristics a right-continuous sample path x_t of the process $\{x_t\}_{t\geq 0}$ starting at x may be constructed as follows. Define $x_t = \Phi(t,x)$ for $0 \leq t < t_1^*$, where t_1^* is the realization of the first jump time t_1 with generalized exponential distribution determined by $P_x[t_1 > t] = \exp\left(-\int_0^t \lambda(\Phi(s,x))ds\right)$. We may now restart the process at $x_{t_1^*}$ according to the same recipe and proceeding recursively we obtain a sequence of jump times $t_1^*, t_2^*, \ldots$ between which X_t follows the integral curve of X. The space S itself is a disjoint union of smooth manifolds S_ι, and jumps happen between different S_ι (the vector field is also parametrized by the index ι). Let us denote now by α the parameter characterizing the point in the classical event–space (it corresponds to ι above). Each observable A of the total system defines now a function $f_A(\psi,\alpha)$ on the space $S = \{(\psi,\alpha)\}$ of pure states of the total system $\Sigma_t = \Sigma_q \otimes \Sigma_c$

$$f_A(\psi,\alpha) = <\psi, A_\alpha \psi> \quad .$$

In [3–8] we showed that the time evolution for observables can be written in a Davis form

$$\begin{aligned}\frac{d}{dt}f_A(\psi,\alpha) &= [(X_H + X_D)f](\psi,\alpha) \\ &\quad +\lambda(\psi,\alpha)\sum_\beta \int Q(\psi,\alpha;d\phi,\beta)[f_A(\phi,\beta) - f_A(\psi,\alpha)]\end{aligned}$$

where Q is the transition probability matrix of the PDP. In Section 3 and 4 we will describe processes associated to semigroups of the above type.

3 BOHM'S VERSION OF THE EPR EXPERIMENT

The anti-realism of the Copenhagen interpretation of Quantum Mechanics was met head – on by the very nice thought experiment proposed 1935 by Einstein, Podolsky and Rosen [16], today commonly called EPR. The argument proceeds by characterizing and using the key notions of Completeness, Reality and Locality. The most perspicuous Bohm version of the EPR argument considers a system which decays into a pair of particles, which travel in opposite directions along the x axis. Ignoring all but spin, each particle, call them L and R for left and right, is associated with its own two dimensional Hilbert space $\mathbf{C}^2$. The spin of the system is zero to start with and this is supposed to be conserved. Thus if L has spin $+1$ in the z direction then R must have spin -1 in the same direction. In the singlet state the system is represented by the state $\psi_{LR} = (|+L> \otimes |-R> + |-L> \otimes |+R>)/\sqrt{2}$ where $|+L>$ and $|-L>$ are the spin eigenstates for the L particle and $|+R>$ $|-R>$ the eigenstates for the R particle. If we measure the spin of the L particle, and if we know that the total spin is conserved, we then know the state of R. While it might be concerned that the spin measurement on the L particle may have disturbed it the same cannot be said of the R particle which should be unaffected by the measurement. In other words we are in a position to predict with probability one the state of the R particle and since we could not have influenced it (Locality) it follows that the spin of the R particle exists independently of measurement (Reality), which implies that Quantum Mechanics, not being able to predict the result with certainty, does not completely describe the whole of reality (Completeness).

Let us describe an EPR-type set up. The quantum Hilbert space $\mathcal{H}_q$ is $\mathbf{C}^2 \otimes \mathbf{C}^2$ (we have two particles). For the classical Hilbert space that accomodates events we choose $\mathbf{C}^3 \times \mathbf{C}^3$ (we have detectors on the left and right).

Let us first remark that the statistics (Bose, Fermi,...) of the two particles that originate from a common source does not play a role here, since in the experimental situation EPR considered the two particles fly apart to the left and right ends of the laboratory. We define now four properties to be measured

E_1: Is spin up in the z-direction at the left end?, $E_2 = E_1^{\perp} = \mathbf{1} - E_1$

E_1': Is spin up in the n-direction at the right end?, $E_2' = E_1'^{\perp} = \mathbf{1} - E_1'$

It is clear that $[E_i, E_j'] = 0, \quad i,j = 1,2.$

In the Hilbert space $\mathcal{H}_t = \mathbf{C}^2 \otimes \mathbf{C}^2 \otimes \mathbf{C}^3 \otimes \mathbf{C}^3$ of the total system we introduce now four operators $V_i, V_j' \quad i,j = 1,2$ defining the Lindblad generator

$$V_i = \sqrt{\lambda}\, E_i \otimes A_i \otimes \mathbf{1}, \qquad V_i' = \sqrt{\lambda'}\, E_i' \otimes \mathbf{1} \otimes A_i,$$

where $(A_i)_{\alpha\beta} = \delta_{i\,\alpha}\delta_{0\,\beta}$, for $i = 1,2\,, \alpha, \beta = 0,1,2$. Assuming quantum Hamiltonian H, it is now easy to describe the associated PD-process. We get the following table of transitions, there rates, probabilities and jumps:

$$\begin{array}{llll}
(0,0) \to (1,0) & \lambda + \lambda' & \frac{\lambda}{\lambda+\lambda'} \|E_1\psi_t\|^2 & \psi_t \to \frac{E_1\psi_t}{\|E_1\psi_t\|} \\
(0,0) \to (2,0) & \lambda + \lambda' & \frac{\lambda}{\lambda+\lambda'} \|E_2\psi_t\|^2 & \psi_t \to \frac{E_2\psi_t}{\|E_2\psi_t\|} \\
(0,0) \to (0,i) & \lambda + \lambda' & \frac{\lambda'}{\lambda+\lambda'} \|E_i'\psi_t\|^2 & \psi_t \to \frac{E_i'\psi_t}{\|E_i'\psi_t\|} \\
(0,1) \to (i,1) & \lambda & \frac{\lambda}{\lambda+\lambda'} \|E_i\psi_t\|^2 & \psi_t \to \frac{E_i\psi_t}{\|E_i\psi_t\|} \\
(1,0) \to (1,i) & \lambda' & \frac{\lambda'}{\lambda+\lambda'} \|E_i'\psi_t\|^2 & \psi_t \to \frac{E_i'\psi_t}{\|E_i'\psi_t\|}
\end{array}$$

where $\psi_t = U(t)\psi = \exp(-iHt)\psi\,.$

We may ask now the question whether the statistics of events on the right depends on measurements performed on the left. To answer this question let us compute the probability that the pointer on the right will jump from 0 to 1 during the time interval $(0,t)$. This can happen in three ways:

1. as a direct transition $(0,0) \mapsto (0,1)$
2. as a composite transition $(0,0) \mapsto (1,0) \mapsto (1,1)$
3. as a composite transition $(0.0) \mapsto (2,0) \mapsto (2,1)$

Transition 1 happens in the time interval $(0,t)$ with probability

$$p_1 = \frac{\lambda'}{\lambda+\lambda'}||E_1'\psi_t||^2 \times \left(1 - e^{-(\lambda+\lambda')t}\right). \tag{3}$$

The composite transition 2 happens with probability

$$\begin{aligned} p_2 &= \int_0^t \frac{\lambda}{\lambda+\lambda'}||E_1U(s)\psi||^2 e^{-(\lambda+\lambda')s}(\lambda+\lambda')ds \\ &\quad \times ||E_1'U(t-s)\frac{E_1U(s)\psi}{||E_1U(s)\psi||}||^2 \times \left(1 - e^{-\lambda'(t-s)}\right) \\ &= \int_0^t \lambda e^{-(\lambda+\lambda')s}ds \times ||E_1'U(t-s)E_1U(s)\psi||^2 \times \left(1 - e^{-\lambda'(t-s)}\right) \end{aligned}$$

and a similar formula with E_1 replaced with E_2 gives p_3. If now

$$[U(s)^\star E_iU(s), U(s')^\star E_j'U(s')] = 0 \qquad s, s' \leq t \tag{4}$$

then a straightforward computation gives the result:

$$p_1 + p_2 + p_3 = ||E_1'U(t)\psi||^2 \left(1 - e^{-\lambda' t}\right). \tag{5}$$

It follows that as long as the usual locality assumptions (4) are satisfied, the event statistics seen on the right does not depend on what is measured on the left, and whether anything is measured there at all. We stress that this observation alone should not be used to conclude that superluminal signalling using EPR is impossible - this for the very reason that we were considering a particular and simplified model. What we proved is only that superluminal communicators must necessarily use more refined methods than the one considered above.

4 CLOUD CHAMBER MODEL AND GRW SPONTANEOUS LOCALIZATION THEORY

Our aim is now to account for the tracks that quantum particles leave in cloud chambers. Physically a cloud chamber is a highly complex system. To describe the response of the chamber to a quantum particle it is sufficient to assume that we have to do with a collection of two state systems able to change their state when a particle passes nearby a sensitive center. Let us sketch the model proposed in [7], [8].

Let us consider the space $E = \mathbf{R}^3$ as filled with a continuous medium which can be at each point $a \in E$ in one of two states: "on" represented by $\binom{1}{0}$ and "off" represented by $\binom{0}{1}$. The set of all possible states of the system is then 2^E. But we will be only interested with a continuum of states namely the "vacuum" and states which differ from the vacuum only in a finite number of points. As vacuum let us choose the state "off" everywhere. The space of classical events can be identified with the space of finite subsets of E from which it follows that the total system

$\Sigma_{tot} = \Sigma_q \otimes \Sigma_c$ is described by families $\{\rho_\Gamma\}_{\Gamma \subset E}$, Γ finite subset of E. For each $a \in E$ let g_a be a Hermitian bounded operator which represents heuristically the sensitivity of the counter located at a. We can think of g_a as a gaussian function $g_a(x)$ centered at $x = a$. We denote

$$\int_E g_a^2(x)da = \Lambda(x). \tag{6}$$

The quantum mechanical Hilbert space is then $\mathcal{H}_q = L^2(\mathbf{R}^3, dx)$. Each state ρ of the total system can be, formally, written as $\rho = \sum_{\Gamma \in \mathcal{S}} \rho_\Gamma \otimes \epsilon_\Gamma$, where, for $\Gamma \in \mathcal{S}$,

$$\epsilon_\Gamma = \prod \otimes_{a \in E} \begin{pmatrix} \chi_\Gamma(a) & 0 \\ 0 & 1 - \chi_\Gamma(a)) \end{pmatrix}, \tag{7}$$

and where χ_Γ stands for the characteristic function of Γ. The Lindblad coupling is now choosen in the following way

$$L_{int}(\rho) \equiv \int_{\mathbf{R}^3} da[V_a \rho V_a - \frac{1}{2}\{V_a^2, \rho\}] \tag{8}$$

where $V_a = g_a \otimes \tau_a, \tau_a$ denoting the flip at the point $a \in \mathbf{R}^3$. Let us introduce the following notation: $a(\Gamma)$ represents the state Γ with the counter at position a flipped, i.e. $a(\Gamma) = (\Gamma \setminus \{a\}) \cup \{\{a\} \setminus \Gamma\}$. The Liouville equation is given by $\dot{\rho} = -i[H, \rho] + L_{int}(\rho)$. But using the following identity in Eq. (8)

$$V_a \rho V_a = \sum_\Gamma g_a \rho_\Gamma g_a \otimes \epsilon_{a(\Gamma)} = \sum_\Gamma g_a \rho_{a(\Gamma)} g_a \otimes \epsilon_a \tag{9}$$

we can write

$$\dot{\rho}_\Gamma = -i[H, \rho_\Gamma] + \int_{\mathbf{R}^3} da g_a \rho_{a(\Gamma)} g_a - \frac{1}{2}\{\Lambda, \rho_\Gamma\} \ . \tag{10}$$

Summing up over Γ we get for the effective quantum state $\hat{\rho} = \sum_\Gamma \rho_\Gamma$

$$\dot{\hat{\rho}} = -i[H, \hat{\rho}] - \int_{\mathbf{R}^3} da g_a \ \hat{\rho} \ g_a - \frac{1}{2}\{\Lambda, \hat{\rho}\} \ . \tag{11}$$

Let us emphasize that the time derivative of $\hat{\rho}$ depends only on $\hat{\rho}$. Moreover (11) is exactly of the type discussed in connection with the spontaneous localization model of Ghirardi, Rimini and Weber [9], the difference being that GRW considered only the constant rate case, and were simply not interested in the classical traces of particles.

We can also construct the associated PD Markov process. We get for time evolution observables the same equation as in (10, except for the sign in front of the Hamiltonian. By taking expectation values we obtain a Davis generator corresponding to rate function $\lambda(\psi) = (\psi, \Lambda\psi)$, and probability kernel Q with non-zero elements of Q given by

$$Q(\psi, \Gamma; d\psi', a(\Gamma)) = \frac{||g_a\psi||^2}{\lambda(\psi)} \delta(\psi' - \frac{g_a\psi}{||g_a\psi||}) d\psi'. \tag{12}$$

Time evolution between jumps is given by:

$$\psi_t = \frac{\exp\left(-iHt - \frac{\Lambda t}{2}\right)\psi_0}{||\exp\left(-iHt - \frac{\Lambda t}{2}\right)\psi_0||} \ . \tag{13}$$

The PD process can be described as follows: $\psi \in L^2(\mathbf{R}^3, dx)$ develops according to the above formula until at time t_1 jump occurs. The jump consists of a pair: (classical event, quantum jump). The classical medium jumps at a with probability density

$$p(a; \psi_{t_1}) = \|g_a \psi_{t_1}\|^2 / \lambda(\psi_{t_1}) \ , \tag{14}$$

(flip of the detector) while the quantum part of the jump is jump of the Hilbert space vector ψ_{t_1} to $g_a\psi_{t_1}/\|g_a\psi_{t_1}\|$ and the process starts again. The random jump time t_1 is governed by the inhomogeneous Poisson process with rate function $\lambda(\psi_t)$. If the medium is homogeneous, then $\lambda(\psi) = \text{const} = \lambda$, and we obtain for quantum jumps the GRW model. More complete discussion can be found in Refs. [7,8]. [2]

Derivation of Born's interpretation Let us consider now the idealized case of a homogeneous medium of particle detectors that are coupled to the particle only for a short time interval $(t, t+\Delta t)$, $\Delta t \to 0$ with intensity λ, so that $\lambda\Delta t \to \infty$. Let us also assume that the detectors are strictly point-like that is, that $g_a^2(x) \to \lambda\delta(x-a)$. In that case the formula (14), giving the probability density of firing the detector at a, becomes $p(a;\psi) = \|g_a\psi\|^2/\lambda = |\psi(a)|^2$ and we recover the Born interpretation of the wave function. The argument above goes as well for the case of a particle with spin.

Remark 1 For free particles in a homogeneous medium, i.e. $H = -\frac{\hbar^2}{2m}\Delta$, and for gaussian wave packets, straight lines are the most probable one. Indeed starting with a moving gaussian wave packet ψ_t then the probability of a registration of the particle at a is $\|f_a\psi_{t_1}\|^2/\lambda$ which is maximum if a coincides with the center of the gaussian.

Remark 2 For different values of the parameters we obtain different situations. Choosing $\lambda \simeq 10^{-9}$ years and a "universal" medium we obtain exactly the spontaneous localization model á la GRW. But we can also obtain standard nice particle tracks. The behaviour depends essentially on the relation between the two time scales: the one given by the energy spectrum and the other provided by the jump rate function.

Remark 3 In [8] the above model is discussed in more details introducing a multiparticle cloud chamber model. For a homogeneous medium one gets, for the effective statistical state of the quantum system, exactly the same equation as in Ref. [18]. For N particles the localization effect is proportional to the number of particles. The rate of jump, even in a homogeneous medium, is no more constant and the formulas given there provide the framework for a numerical simulation.

Remark 4 Several authors tried to explain track formation by a pure Hamiltonian theory (for recent attempts cf. Ref [19,20]). We know of no successful derivation that leads to a clear law relating rate of detections to geometry of detector locations and shape of wave function. The law we derive could have been in principle obtained already by E.B. Davies [21].

[2] We could translate our simple algorithm of track formation into the language of stochastic differential equations and filtering theory (cf. e.g. Ref. [17]), but that would serve no useful purpose at all – as all questions of interest can be answered using PDP algorithm, either by analytic computation, or by numerical simulation as in Monte Carlo Wave Function method of Refs. [12–15].

CONCLUDING REMARKS

We have seen that the word "measurement" instead of being banned, as suggested by J. Bell [22,23] can be given a precise and acceptable meaning: an appropriate CP coupling between the quantum system and a classical event-system, where information about the quantum states is transfered to the classical recording device by a continuous family of CP maps of the total system. It is fundamental to note that a transfer of information from Σ_q to Σ_c is impossible by any automorphic evolution. For a discussion of this fact see [6] and also Landsmann [24] and the no-go theorem by Ozawa [25]. In the framework we propose, all probabilistic properties of Quantum Mechanics – as e.g. Born's interpretation of the scalar product as a probability amplitude – can be derived thanks to the PD Markov process.

Our approach is in some respects similar in spirit to the idea of Quantum Stochastic Processes of E.B. Davies [26], especially to his class of "transition processes". The main difference being that he took the space of events as a primary object, without recognizing usefulness of introducing a classical system whose states can parametrize quantum dynamics and jump rates, and whose changes of states constitute events.

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