HTML ABSTRACT + LINKS

Some Comments on the Formal Structure of Spontaneous Localization Theories

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Abstract. We propose a mathematical and a conceptual framework that encompasses and generalizes the "flash" ontology discussed in a recent paper by R. Tumulka [1].

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INTRODUCTION

Tumulka [1] proposed a generalization of the GRW model [2] (for a recent overview see [3]) and a new ontology for discussing quantum phenomena, an ontology that he called "the flash ontology". We propose a method of generalizing Tumulka's idea even further, so as to include a class of models for which the arena where the "flashes" take place may be different than 3-space. For instance, our proposed generalized framework may well include models avoiding explosion of energy as discussed recently by Bassi et al. [4], as well as models described in finite-dimensional Hilbert spaces (e.g. pure spin models) or models where "flashes" are accompanied by phase transitions. We will also argue that the flash ontology can be traced back to a more general "event ontology". At the end we will discuss possible advantages of such a generalization and discuss some applications.

For the purpose of this paper we will call Tumulka's model the "F-model" (F - for flash), and we will call our proposed generalization the "E-model" (E - for event). Loosely speaking "events" are more general than flashes. While every flash constitutes an event, we can think of events (for instance receiving a bit of information) that are not accompanied by flashes. It should be mentioned though that in [1] both terms, flashes and events, are being used, so our distinction between the two is rather of an organizing than of a principal significance.

A BRIEF DESCRIPTION OF THE F-MODEL

We will describe the model for one particle. Extending it to many particles, or two quantum field theory, does not represent essentially new conceptual or technical problems (cf. "Concluding Remarks" at the end of this paper). Let $\mathfrak{H}$ be the Hilbert space $L^2(\mathbb{R}^3)$, and let H be a self–adjoint Hamilton operator in $\mathfrak{H}$ defining the evolution of the particle when the flash rate is zero. The main object in Tumulka's F-model is a positive operator valued function $\Lambda(\mathbf{x})$, $\mathbf{x} \in \mathbb{R}^3$. The physical dimension of $\Lambda(\mathbf{x})$ should be $1/(\text{time}\times\text{space})$. For each Borel set $B \in \mathbb{R}^3$, $\Lambda(B)$ denotes the integral of $\Lambda(\mathbf{x})$ over B, in particular $\Lambda(\mathbb{R}^3)$ is

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the integral of $\Lambda(\mathbf{x})$ over the whole space.[1] The evolution of the wave function between flashes is given by the evolution operator

$$W_t = \mathrm{e}^{-\frac{1}{2}\Lambda(\mathbb{R}^3)t - \mathrm{i}Ht}. \tag{1}$$

After each flash, assuming that the flash happened at the point $\mathbf{X}$, the wave function "collapses":

$$\psi \mapsto \frac{\Lambda(\mathbf{X})^{1/2}\psi}{\|\Lambda(\mathbf{X})^{1/2}\psi\|}. \tag{2}$$

The joint probability distribution of the first n flashes to happen at space–time points $(T_1, \mathbf{X}_1), \ldots, (T_n, \mathbf{X}_n)$ is postulated to be (cf. Ref. [1, Eq. (11)]:

$$\begin{aligned}\mathbb{P}\big(\mathbf{X}_1 \in \mathrm{d}^3\mathbf{x}_1, T_1 \in \mathrm{d}t_1, \ldots, \mathbf{X}_n \in \mathrm{d}^3\mathbf{x}_n, T_n \in \mathrm{d}t_n\big) &= \\ \big\|K_n(0, \mathbf{x}_1, t_1, \ldots, \mathbf{x}_n, t_n)\,\psi\big\|^2\, \mathrm{d}^3\mathbf{x}_1\,\mathrm{d}t_1 \cdots \mathrm{d}^3\mathbf{x}_n\,\mathrm{d}t_n\,, \end{aligned} \tag{3}$$

where K_n is an operator-valued function defined by

$$\begin{aligned}K_n(t_0, \mathbf{x}_1, t_1, \ldots, \mathbf{x}_n, t_n) &= \\ \Lambda(\mathbf{x}_n)^{1/2}\, W_{t_n - t_{n-1}}\, \Lambda(\mathbf{x}_{n-1})^{1/2}\, W_{t_{n-1} - t_{n-2}} \cdots \Lambda(\mathbf{x}_1)^{1/2}\, W_{t_1 - t_0}\,. \end{aligned} \tag{4}$$

The flashes are supposed to be objective space–time events. Tumulka writes:

> "It is tempting to regard the collapsed wave function ψ_t as the ontology, but I insist that the flashes form the ontology."

THE E–MODEL

From F-model to E-model

The E–model can be thought of as a natural and far reaching extension of the F–model. The concept of a "flash" is replaced by that of an "event", with the idea that the model can be applied to more general systems than systems of particles in space–time. For instance it may used in quantum cosmology and applied there to the whole universe. The Big Bang may be such an "event", accompanied by some kind of a wave collapse, possibly with phase transition. As in the F-ontology, so here events are thought to be objective and therefore they can be recorded. It is, in principle, possible to analyze the recorded data and to analyze the past history of the events. It is possible to adjust the quantum evolution at each given time t, depending on the observed events till the time t. Moreover, as we will see, it costs us nothing to allow for phase transitions

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[1] A similar object has been introduced in Ref. [5], but care must be taken when comparing objects denoted by the same symbols in Refs. [1] and [5]. $\Lambda^{1/2}(\mathbf{a})$ of [1] corresponds to the operator of multiplication by the function $f_{\mathbf{a}}(\cdot)$ of [5], while $\Lambda(\mathbb{R}^3)$ of Ref. [1] corresponds to the operator denoted simply by Λ in [5] - cf. [5, Eq. (19)].

that can accompany some of the events. Formally such a phase transition may involve change of the Hamiltonian, or even change of the underlying Hilbert space. Last but not least, we will allow for some sample histories to have no events at all - this may happen for instance when an elementary particle passes through a particle detector. It may well happen that it will cause no detection event at all, due to the limited detector sensitivity. While the F-model has been constructed with the purpose of reproducing and generalizing the GRW spontaneous localization mechanism, the E-model extends the formal framework of the F-model and enhances it, allowing for a wider scope of applications, beyond just space–time collapses. The fact that we allow for dependence of the current evolution on the past history of events may suggest that we are going beyond the Markovian models. But this is not the case. It is enough to introduce the recording medium as a part of the system, and then the evolution becomes Markovian again, while at the same time adding flexibility and allowing for a wider scope of applications. We will now introduce the E-model in its full generality, and later on describe how the F-model becomes a particular case of the F-model, and how the flash rate density postulate (3) of Ref. [1] can be derived (rather than postulated) from a simple and natural, Lindblad's type, Master equation.

The formal structure of the E-model

Let S be a set. Heuristically S is to be thought of as "the set of all the potential states of the event recording medium". S may have any cardinality required, but for the sake of simplicity we will assume in this Section that S is a finite set. Generalization to more complicated case (such as, for instance, the GRW model, or the F-model of Ref. [1]) is straightforward: it requires replacing sums by integrals etc. Elements of S will be denoted by Greek letters α, β, etc. For every $\alpha \in S$ let there be given a Hilbert space $\mathfrak{H}_\alpha$. In most of the applications (when there are no phase transitions) all Hilbert space $\mathfrak{H}_\alpha$ can be identified: $\mathfrak{H}_\alpha \equiv \mathfrak{H}, \forall \alpha \in S$. For each $\alpha \in S$ let $H_\alpha(t)$ be a self-adjoint Hamiltonian operator acting in $\mathfrak{H}_\alpha$. For the sake of generality we allow H_α to depend explicitly on time $t \in \mathbb{R}$. We define an "event" to be an ordered pair (α, β), $\alpha \neq \beta$, $\alpha, \beta \in S$. Thus, heuristically, an event is defined as a "change of state of the event recording medium" – a natural definition. The final piece of data consists in associating to each event $\alpha \to \beta$ a transition operator $G_{\beta\alpha} : \mathfrak{H}_\alpha \longrightarrow \mathfrak{H}_\beta$. For convenience we will extend the definition of $G_{\alpha\beta}$ to the case of "non-events", $\beta = \alpha$ by putting $G_{\alpha\alpha} \doteq 0, \forall \alpha$. In Ref.[1] an event is a flash at $\mathbf{X} \in \mathbb{R}^3$, and the associated jump (or "collapse") operator is $\Lambda(\mathbf{X})^{\frac{1}{2}}$. Here the transition operators are the primitive objects, while the positive operators Λ are composite. The relation between G-s and Λ-s is many–to–one. It is important to notice that it is, in general, not necessary for these transition (or "jump") operators to be positive, and we will allow for this more general case in our model. As with the Hamiltonians $\mathfrak{H}_\alpha$, so with the jump operators, we can allow $G_{\beta\alpha}(t)$ to depend explicitly on time. Assuming the initial state is described by a vector $\psi_\alpha \in \mathfrak{H}_\alpha$, the sample history will be piecewise deterministic: there will be a continuous evolution $\psi_\alpha(t)$ between jumps, interrupted with a sequence of transitions $\alpha_k \to \alpha_{k+1}$ at discrete times T_k, accompanied by quantum jumps $\mathfrak{H}_{\alpha_k} \ni \psi_k(T_k) \to \psi_{k+1}(T_k) \in \mathfrak{H}_{\alpha_{k+1}}$. Averaging over these discrete, random jumps, the initial pure state ψ_α will evolve continuously into a mixture state with components in different $\mathfrak{H}_{\alpha_k}$. Let $\rho = \{\rho_\alpha\}$, $\rho_\alpha \geq 0 \forall \alpha$, $\mathrm{Tr}(\rho) = \sum_\alpha \mathrm{Tr}(\rho_\alpha) = 1$, be the density matrix of our system. Then we postulate the following evolution equation as the basic equation for the E–model:

$$\frac{\mathrm{d}\rho_\alpha}{\mathrm{d}t} = -i[H_\alpha, \rho_\alpha] + \sum_\beta G_{\alpha\beta}\, \rho_\beta\, G^\star_{\alpha\beta} - \frac{1}{2}\{\Lambda_\alpha, \rho_\alpha\}, \tag{5}$$

where

$$\Lambda_\alpha = \sum_\beta G^\star_{\beta\alpha} G_{\beta\alpha}, \tag{6}$$

and G_α, H_α may depend explicitly on t. By taking the trace and summing over α we find that $\mathrm{Tr}(\rho)$ is conserved. Moreover, Eq. (5) is of Lindblad's type, thus positivity is preserved as well. While Eq. (5) describes the average behavior of a statistical ensemble, a typical sample history is described by a simple piecewise deterministic Markov process described in what follows. Let us introduce the following notation:
For each $\alpha \in S$ and each pair $t_0 < t$ of time instants, let $W_\alpha(t, t_0)$ be the unique solution of the differential equation:[2]

$$\dot{W}_\alpha(t, t_0) = \left(-\mathrm{i}H_\alpha(t) - \frac{1}{2}\Lambda_\alpha(t)\right) W_\alpha(t, t_0), \tag{7}$$

with the initial condition $W_\alpha(t, t_0)|_{t=t_0} = I_\alpha$, the identity operator on $\mathfrak{H}_\alpha$. For each $t \in \mathbb{R}$, $\beta \neq \alpha$, $\psi \in \mathfrak{H}_\alpha$, let

$$p_{\alpha\to\beta}(\psi, t) = \frac{\langle \psi | G_{\beta\alpha}(t)^\star G_{\beta\alpha}(t)\psi\rangle}{\langle \psi, \Lambda_\alpha(t)\psi\rangle} = \frac{\|G_{\beta\alpha}(t)\psi\|^2}{\sum_\gamma \|G_{\gamma\alpha}(t)\psi\|^2}. \tag{8}$$

The sample path of the unique piecewise deterministic Markov process reproducing Eq. (5) is then described as follows:

Given on input t_0, α_0, and $\psi_0 \in \mathfrak{H}_{\alpha_0}$, with $\|\psi_0\| = 1$, it produces on output t_1, α_1 and $\psi_1 \in \mathscr{H}_{\alpha_1}$, with $\|\psi_1\| = 1$.

1) Choose uniform random number $r \in [0, 1]$.
2) Propagate ψ_0 in $\mathfrak{H}_{\alpha_0}$ forward in time:

$$\psi(t) = W_\alpha(t, t_0)\psi_0 \tag{9}$$

until $t = t_1$, where t_1 is defined by[3]

$$\|\psi(t_1)\|^2 = r. \tag{10}$$

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[2] We disregard possible complications coming from the necessity of taking care of the domains of definition of the involved generators

[3] Note that, as can be seen from the equation (7), due to non–negativity of the operators $\Lambda_\alpha(t)$, the norm of $\psi(t)$ is a monotonically decreasing function of t.

3) Select the new state $\alpha_1 \in S$, among the states $\alpha \neq \alpha_0$, using the probability distribution $p_{\alpha_0 \to \alpha}\left(\psi(t_1), t_1\right)$.

4) Set
$$\psi_1 = G_{\alpha_1\alpha_0}(t_1)\,\psi(t_1)/\|G_{\alpha_1\alpha_0}(t_1)\,\psi(t_1)\| \in \mathfrak{H}_{\alpha_1} \tag{11}$$
. Goto 1) replacing α_0 with α_1, t_0 with t_1 and ψ_0 with ψ_1.

The sample history of an individual system is described by a repeated application of the above algorithm, using its output as the input for each next step.

As a Corollary we easily get the following joint distribution of the first n events:

$$\mathbb{P}\big(\alpha_1, T_1 \in \mathrm{d}t_1, \ldots, \alpha_n, T_n \in \mathrm{d}t_n\big) = \\ \big\|K_n(\alpha_0, t_0, \alpha_1, t_1, \ldots, \alpha_n, t_n)\,\psi_0\big\|^2\,\mathrm{d}t_1 \cdots \mathrm{d}t_n\,, \tag{12}$$

where K_n is an operator-valued function on $(S \times \text{time})^{n+1}$, $K_n : \mathfrak{H}_{\alpha_0} \to \mathfrak{H}_{\alpha_n}$, defined by

$$K_n(\alpha_0, t_0, \alpha_1, t_1, \ldots, \alpha_n, t_n) = \\ G_{\alpha_n\alpha_{n-1}}(t_n)\,W_{\alpha_{n-1}}(t_n, t_{n-1})G_{\alpha_{n-1}\alpha_{n-2}}(t_{n-1})\,W_{\alpha_{n-2}}(t_{n-1}, t_{n-2}) \cdots G_{\alpha_1\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,. \tag{13}$$

The derivation of Eq. (12) from the Markov process is straightforward. According to step 1) the process resulting in the first jump is an inhomogeneous Poisson process with rate function
$$\lambda(t) = \frac{\langle W_{\alpha_0}(t, t_0)\,\psi_0|\Lambda_{\alpha_0}(t)W_{\alpha_0}(t, t_0)\,\psi_0\rangle}{\langle W_{\alpha_0}(t, t_0)\,\psi_0|W_{\alpha_0}(t, t_0)\,\psi_0\rangle}. \tag{14}$$
The probability of surviving without any event until t_1 is $\|W_{\alpha_0}(t_1, t_0)\|^2$, and the probability of an event during the time interval t_1 and $t_1 + \mathrm{d}t$ is $\lambda(t_1)\mathrm{d}t$, so that the probability that the first jump will occur between t_1 and $t_1 + \mathrm{d}t_1$ is

$$\|W_{\alpha_0}(t_1, t_0)\|^2 \times \lambda(t_1)\mathrm{d}t = \langle W_{\alpha_0}(t_1, t_0)\,\psi_0|\Lambda_{\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,\psi_0\rangle \mathrm{d}t.$$

Now, the probability of a transition $\alpha_0 \to \alpha_1$ at t_1 is

$$p_{\alpha_0 \to \alpha_1}(t_1, W_{\alpha_0}(t_1, t_0)\,\psi_0) = \frac{\|G_{\alpha_1\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,\psi_0\|^2}{\langle W_{\alpha_0}(t_1, t_0)\,\psi_0|\Lambda_{\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,\psi_0\rangle},$$

and so the probability that the first event will happen between t_1 and $t_1 + \mathrm{d}t$, and will be accompanied by the transition $\alpha_0 \to \alpha_1$ is the product

$$\langle W_{\alpha_0}(t_1, t_0)\,\psi_0|\Lambda_{\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,\psi_0\rangle \mathrm{d}t \times \frac{\|G_{\alpha_1\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,\psi_0\|^2}{\langle W_{\alpha_0}(t_1, t_0)\,\psi_0|\Lambda_{\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,\psi_0\rangle} = \\ = \|G_{\alpha_1\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,\psi_0\|^2 \mathrm{d}t. \tag{15}$$

Repeating the steps, starting now with

$$\psi_1 = \frac{G_{\alpha_1,\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,\psi_0}{\|G_{\alpha_1,\alpha_0}(t_1)W_{\alpha_0}(t_1, t_0)\,\psi_0\|}$$

leads to equations (12) and (13).

FROM E–MODEL TO F–MODEL

A sequence of specializations leads from the E–model described above to the F–model of Ref. [1]:

1. The set S is specialized to be the set of all finite sequences $\{\mathbf{x}_1,\ldots,\mathbf{x}_n\}$, $\mathbf{x}_i \in \mathbb{R}^3$. If α and β are two such sequences, then $G_{\beta\alpha} \neq 0$ only if β can be obtained from α by adding one space point at the end: $\alpha = \{\mathbf{x}_1,\ldots,\mathbf{x}_n\}$ and $\beta = \{\mathbf{x}_1,\ldots,\mathbf{x}_n,\mathbf{x}_{n+1}\}$. An event $\alpha \to \beta$ is then interpreted as a "flash" at $\mathbf{x}_{n+1}$.
2. Assume that all Hilbert spaces are identical $\mathfrak{H}_\alpha = \mathfrak{H}$, that there is only one, time–independent Hamiltonian $H_\alpha(t) = H$, and that the transition operators do not depend neither on the initial point $\alpha \in S$, nor on time t: $G_{\alpha,\alpha_0}(t) = G_\alpha(t)$. Notice then if this is the case, then $\Lambda_\alpha(t) = \sum_\beta G_\beta^\star G_\beta \equiv \Lambda$ does not depend neither on the index α nor on time t. The operators $W_\alpha(t,t_0)$ do not depend on α, they depend only on the difference $t-t_0$ and are given by $W_\alpha(t,t_0) = W_(t-t_0)$, where

$$W_t = \mathrm{e}^{\left(-\mathrm{i}H-\frac{1}{2}\Lambda\right)t}$$

   . Notice that, with these assumptions, introducing $\rho = \sum_\alpha \rho_\alpha$, the Master equation (5) can be now summed over α and leads to the evolution equation for ρ:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \sum_\beta G_\beta\,\rho, G_\beta^\star - \frac{1}{2}\{\Lambda,\rho\},. \tag{16}$$

3. With $\alpha = \{\mathbf{x}_1,\ldots,\mathbf{x}_n\}$ let us furthermore assume that G_α depends only on the last element of the sequence $G_{\{vx_1,\ldots,\mathbf{x}_n\}} = G_{\mathbf{x}_n}$. Assume, moreover, that the operators $G_\mathbf{x}$ are non–negative.

With the above specializations, identifying $G_\mathbf{x}^2$ of the E–model with $\Lambda(\mathbf{x})$ of the F-model, and replacing the sum in Eq. (12) by the integral, we recover the joint probability distribution, and thus the flash generating process of Ref. [1].

CONCLUDING REMARKS

Tumulka begins his paper [1] discussing the flash ontology with the following statement:

> John S. Bell concluded from the quantum measurement problem that "either the wave function, as given by the Schrödinger equation, is not everything or it is not right" [7]. Let us assume, for the purpose of this paper, the second option of the alternative [...]

But, in fact, the flash ontology, and to even greater extent the event ontology discussed above require the first option as well: the wave function is not everything. Flashes, and more generally and more distinctively, events form a separate ontology.

Possibility of a chaotic behavior

In general the operators $\Lambda(\mathbf{x})$ of Ref. [1] do not have to form a commutative family. They commute for a GRW model, where they are all functions of the standard quantum–mechanical position operator of the particle. But, for instance, Bassi et al. [4] suggested models in which the transition operators can be functions of the position *a*nd momentum operators. Cases with non–commuting transition operators have been studied numerically within a simple class of E–models - cf. Ref [6], where it was shown that the resulting patterns of events may show a chaotic, fractal–like behavior.

Taming the energy increase

While I was demonstrating a computer simulation of the flash generating process during the recent conference in honor of GianCarlo Ghirardi's 70th birthday, Roderich Tumulka pointed to me that the average time distance between subsequent flashes seems to be getting smaller and smaller. I did not notice it before, so after the conference has ended I wrote a specialized computer program to investigate just this phenomenon. Lo and behold, Tumulka was right. It is only then that I have discovered that the phenomenon is known to the experts, and that the recent paper of Bassi at al. [4] emerged from the discussion of this phenomenon, and from the attempt to provide a cure for the possible infinite increase of the velocity of the particle as measured by the frequency of flashes. The cure seems to consist of mixing space and momentum localizations. Such a generalization is available within the E–model. Replacing space by phase space, and replacing multiplication operators by Gaussian functions of $\mathbf{x}$, as in GRW model, with Wigner quantized Gaussian functions of the position $\mathbf{x}$ and the momentum $\mathbf{p}$ may lead to a model along the lines indicated in [4]. Wigner's quantization does not preserve positivity, but positivity of the transition operators is not required in the E–model. The fact that the resulting operators will then form a non–commuting family may lead to an extra chaotic behavior of the observed trajectories.

Quantum Field Theory

Generalization of the proposed framework to the case of many particles (whether distinguishable or not), and then further to quantum field theory, is straightforward as it has been shown in Refs. [5] and [1].

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